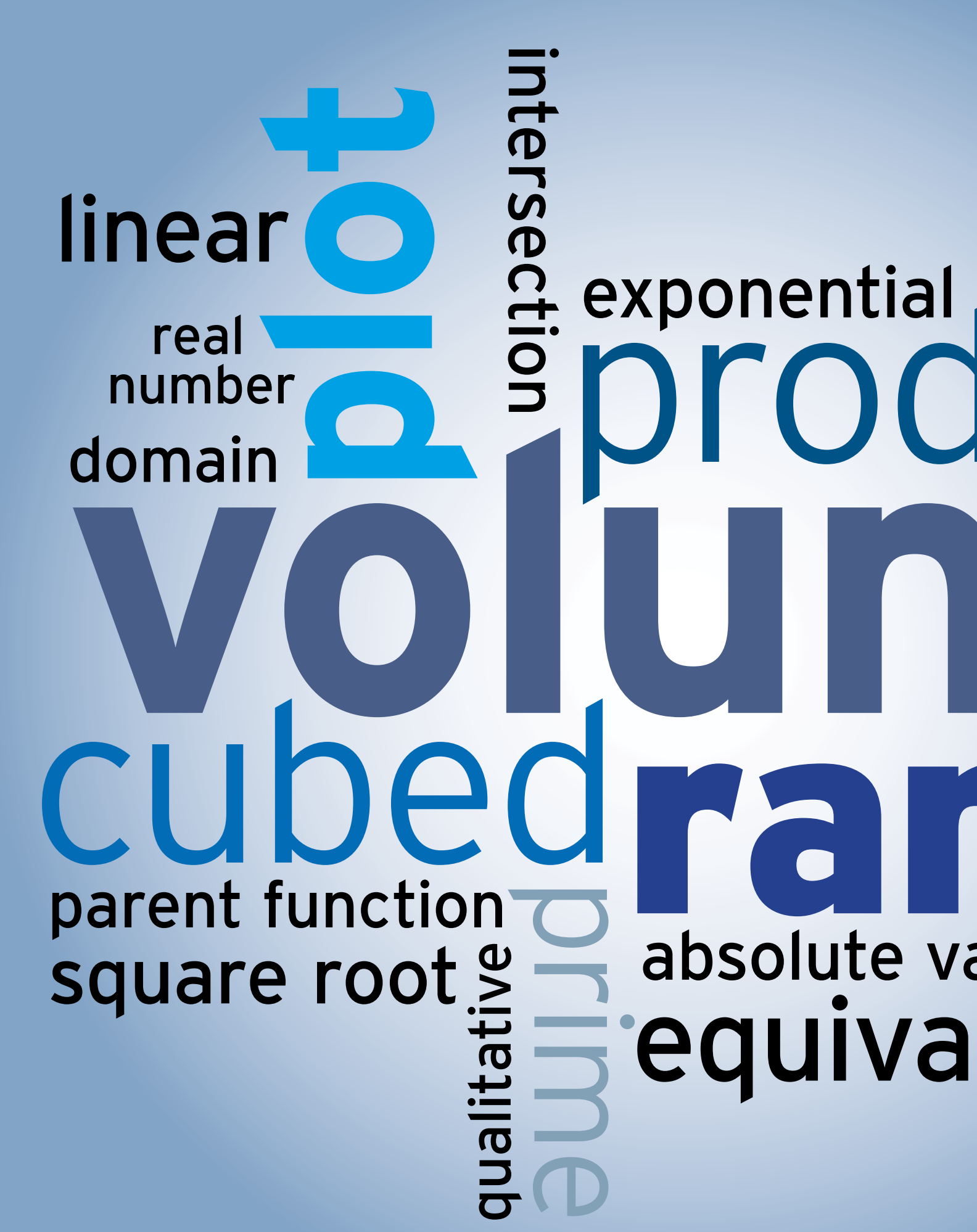

For ELLs: Vocabulary beyond the Definitions

A classroom teacher discusses ambiguities in mathematics vocabulary and strategies for ELL students in building understanding.

Nancy S. Roberts and Mary P. Truxaw

"But where is the exponent?" Jorge, a tenth-grade English language learner (ELL), asked me while I (co-author Roberts) was talking about the formula for the area of a parallelogram. After much confusion on my part, Jorge said, "Last year you said that the base was the number in a power that was not the exponent. I don't see the exponent, so I don't know where the base is." Aha! I *had* said something like that in algebra class the previous year. However, I had never thought about the two different uses of the word *base* within mathematics.

Although I knew some of the challenges faced by ELL students learning mathematics vocabulary, I had never considered that mathematics, known for its precision, would include ambiguity *within* its vocabulary. In fact, the sixth Standard for Mathematical Practice within the Common Core State Standards for Mathematics (CCSSM) relates to attending to precision: "Mathematically proficient students" need to "communicate precisely to others" and "try to use clear definitions in discussion with others and in their own reasoning" (CCSSI 2010, p. 7).

I thought about Jorge. He had been confident enough and had the language skills to ask for clarification; many ELL students might not. If I had not recognized the connection to my earlier use of mathematics vocabulary, where would this

confusion have led? How would I have uncovered it? How would Jorge's confusion have impeded mastering important mathematical practices or communicating precisely? These questions and others led to my investigation into the role of vocabulary development in helping ELL students be successful in mathematics, specifically in first-year algebra.

As I considered the importance of supporting ELL students' mathematics vocabulary, I asked myself a question that would likely arise for many mathematics teachers: "Do I have time to spend on vocabulary development?" Jorge helped me recognize that I had to ask myself a different question: "Can I afford *not* to spend time on vocabulary development?" Many vocabulary strategies that have worked for my students do not add much additional time and enhance not only vocabulary but also the mathematics.

CHALLENGES OF MATHEMATICS VOCABULARY FOR ELLS

Although mathematics language is much more than just learning vocabulary (Moschkovich 1999, 2002; NCTM 2000), vocabulary development is still central to learning to read, write, speak, listen to, and make sense of mathematics (CCSSI 2010; Heinze 2005). I will focus specifically on helping ELL students build better understanding of algebra through vocabulary, sharing outcomes of my own learning about mathematics vocabulary and strategies that worked for my students and me.

Mathematics vocabulary may be more difficult to learn than other academic vocabulary for several reasons.

1. Definitions are filled with technical vocabulary, symbols, and diagrams (Pimm 1987). Teachers need to explicitly help students make sense of this new language (Schlepegrell 2007).
2. Many mathematics concepts can be represented in multiple ways. At least thirteen different terms can mean subtraction (Echevarria, Vogt, and Short 2010; Heinze 2005). Multiplication can be indicated in many ways: "2 times 3," "2 multiplied by 3," and "the product of 2 and 3." To add to the confusion, some words may have similar connotations but vastly different technical meanings—for example, "3 multiplied by 10" and "3 increased by 10" (Heinze 2005).
3. Many mathematics words have multiple meanings. A *quarter* may refer to a coin or a fourth of a whole. Students must learn that the same word in different situations has different meanings, such as asking for a quarter while at a vending machine or while eating a pizza (Moschkovich 2002).
4. The overlap between mathematics vocabulary and everyday English (Kotsopoulos 2007; Moschkovich 2002) is problematic (see **table 1**). The word *product*, for instance, has meaning in everyday English that is completely different from its very specific mathematical meaning.
5. Homonyms and words that sound similar can confuse (Adams 2003). See **table 2** for a partial list.
6. Similarity to native language words can add more confusion. Although these similarities may sometimes be helpful—as when cognates have similar sounds and similar meanings—similarities can also contribute to confusion. For example, the Spanish word for *quarter* is *cuarto*, which can mean "a quarter of an hour"; *quarter* could also mean a room in a house, as in the English usage "your living quarters" (Moschkovich 1999, 2002).

Table 1 Math Usage vs. Everyday Usage

Vocabulary Word	Mathematics Usage	Everyday Usage
volume	Amount of space	Noise level
product	Result in multiplying	Item produced in manufacturing
plot	To graph a point	A piece of land to build a house
cubed	Raised to the third power	A type of steak or a way to cut vegetables
range	Numerical difference between two values	Stove top
prime	Prime number	Prime rib, prime time

Source: Adams (2003), p. 789

Table 2 Homonyms and Similar Sounding Words

whole – hole	eight – ate	sum – some
two – to – too	symbol – cymbal	sides – size
tenths – tents	half – have	real – reel

Source: Adams (2003), p. 790–91

Clearly, vocabulary is an important issue in mathematics classrooms, especially for ELL students.

TEACHING METHODS AND STRATEGIES

A selection of strategies for supporting students' development of mathematics vocabulary and examples of how to use them follow. Suggestions illustrate vocabulary support within an algebra unit but could be adapted for other topics. Two tools that will be highlighted are word walls—organized collections of words displayed in the classroom to support vocabulary development—and graphic organizers—visual charts and representations designed to organize student learning. We will also look at ways in which these tools can encompass vocabulary strategies.

Table 3 Sample Algebra Vocabulary					
absolute value	binomial	coefficient	complete the square	conjunction	derive
domain	equivalent	exponential	function	intersection	interval
inverse	linear	monomial	parabola	parent function	piecewise
polynomial	qualitative	quadratic	radicand	range	rational
real number	regression	solution	square root	trinomial	variable

Source: CCSSI (2010), pp. 52–71

Develop a Vocabulary List

Begin by developing a vocabulary list for the unit. **Table 3** shows samples of mathematics vocabulary from the Common Core State Standards for algebra (CCSSI 2010). Along with traditional algebra terms, include vocabulary to support challenges for ELLs, as described earlier (e.g., *symbol* and *whole*). Scaffolding such as word walls and graphic organizers will increase vocabulary usage while reducing cognitive load and stress (Echevarria, Vogt, and Short 2010).

Preteach and Assess

At the start of a unit, it is beneficial to trigger and assess prior knowledge, review previously learned vocabulary, and preteach new vocabulary. Preteaching vocabulary words requires explicit teaching of definitions, pronunciation, and word parts (Paulsen 2007).

Word Walls

One strategy for stimulating and assessing prior knowledge is a word wall. At the beginning of the unit, display all the vocabulary for the unit to act as an *anticipation guide*, a strategy used during preteaching to stimulate interest in a topic and give students a preview of what is to come. One way to use a word wall as a preassessment tool and as the trigger on the first day of a unit is to include a word that does not belong. Then ask small groups to pick out the word and describe why it does not belong. In a graphing unit, for example, the word wall could include the term *scientific notation* along with graphing words such as *slope*, *y-intercept*, *ordered pair*, *xy-intercepts*, and so on. (The nonconforming word would later be removed from the word wall.)

Another way to use word walls for preassessment is to have students organize the words into groups and give reasons for their choices. Words relating to a unit on exponents might be *base*, *exponent*, *denominator*, *numerator*, *polynomial*, *monomial*, *binomial*, *trinomial*, *power*, *reciprocal*, *coefficient*, and *factor*. One student might group *denominator*, *numerator*, and *reciprocal* as words related to fractions; another student might group *base*, *exponent*, and *power* as words describing a power. Listening to discussions provides interactive forms of preassessment. Moreover, student explanations provide opportunities to foster CCSS mathematical practices—for example, communicating precisely to others and constructing viable arguments.

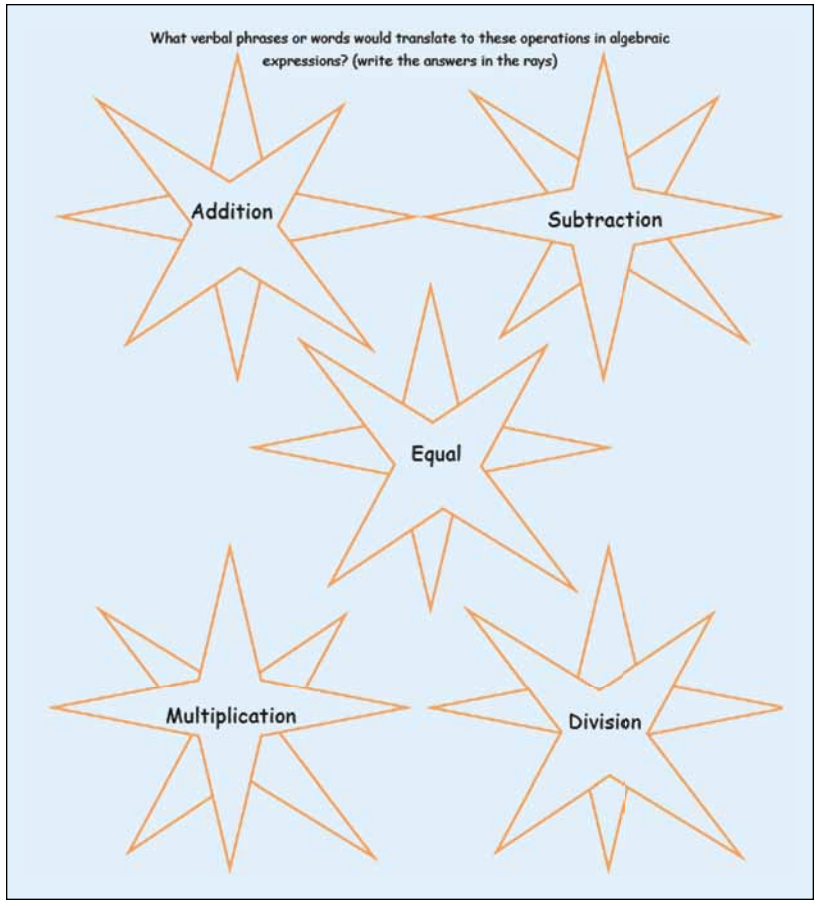

Fig. 1 The points of the stars provide space for students to write phrases that mean the same thing.

Graphic Organizers

Graphic organizers can be useful for activating and assessing students' prior knowledge, organizing different ways to express basic mathematical concepts, and organizing vocabulary for long-term retention. One organizer includes eight-sided stars with words for arithmetic operations and equality (see **fig. 1**). Working with partners, students list

words that could be used for each operation. Then they add to their lists by comparing these in small groups. Finally, the class as a whole reviews the words. This class review is a time to make connections to the mathematical concepts, to address misconceptions, and to include words and phrases that are often confusing—for instance, "4 less than x" to mean "x minus 4."

Teach and Reteach

Researchers have provided many suggestions for explicitly teaching and reteaching vocabulary (see, e.g., Adams 2003; Gee 1996; Moschkovich 2002; Paulsen 2007). The focus here will be on word walls and graphic organizers.

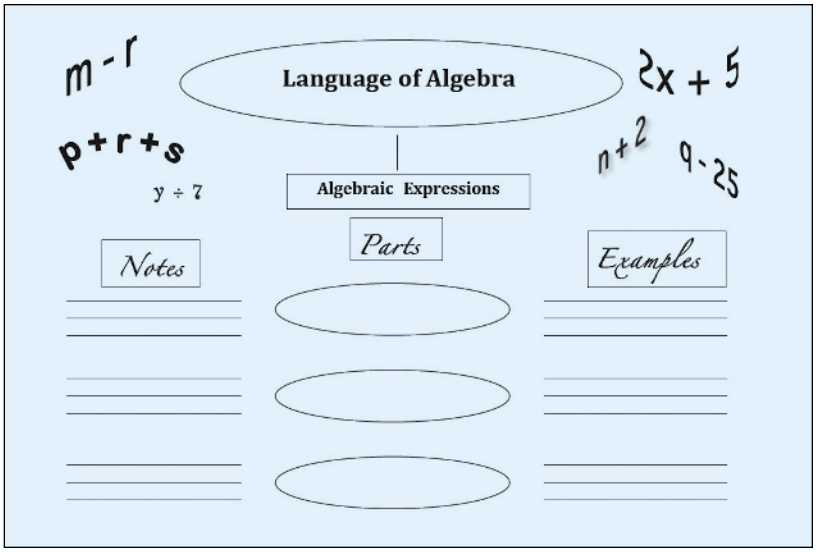

Fig. 2 *Variable*, *constant*, and *operation* would be appropriate entries in the ovals in the middle column.

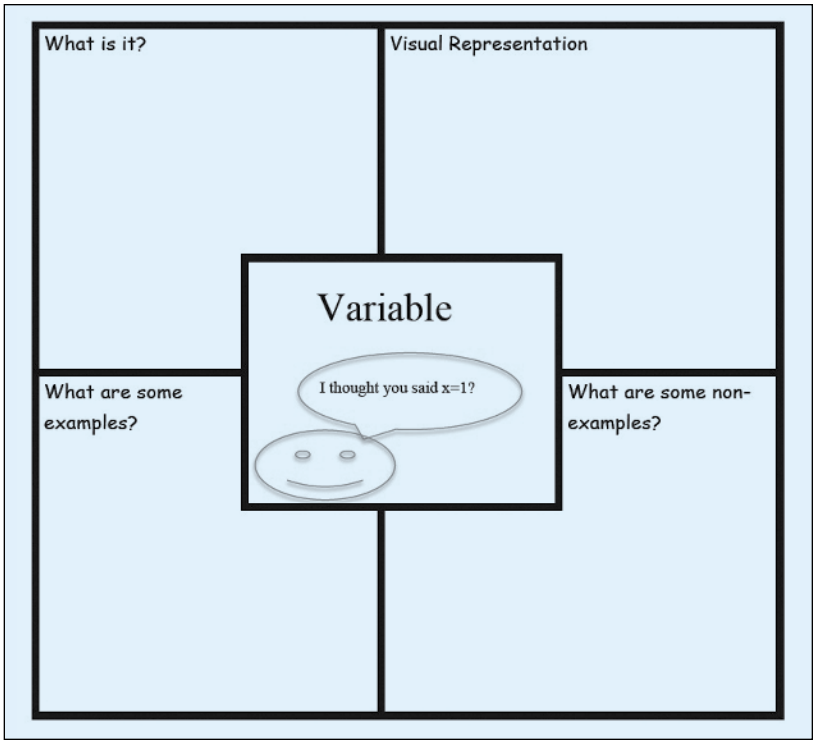

Fig. 3 A Frayer model is useful for some vocabulary.

Word Walls

Word walls are also useful within instructional units. A key idea is that word walls should be interactive, not static. After explicitly teaching words in the context of the unit, add definitions, examples, and diagrams to the words on the wall. Using nonexamples can help refine or clarify definitions (Adams 2003). In addition, real-life situations can provide context for algebra vocabulary and concepts (Paulsen 2007).

A helpful strategy is to start with informal definitions (while preteaching and assessing prior knowledge) and then transition to formal definitions (NCTM 2000). For example, the informal definition "a variable is a letter" may lead to "a variable is a symbol that represents a number" and finally to "a variable is a symbol, usually a letter, that is a quantity that can have different values." Informal definitions help students construct their own meaning, but formal definitions help them understand and apply concepts presented in mathematics textbooks (Adams 2003).

Ongoing interactive use of the word wall helps students see its value. As the year progresses, students use the word wall when answering verbal questions, when writing responses to essential questions on tests, and at other times when vocabulary usage is emphasized.

Graphic Organizers

Graphic organizers are beneficial within a unit of study to build and reinforce mathematics language. A graphic organizer entitled The Language of Algebra provides an opportunity to teach or reteach the parts of an algebraic expression by giving definitions and examples in the context of expressions (see **fig. 2**). In this specific organizer, the "parts" section (middle column) could list *variable*, *constant*, and *operation*, with notes and examples for each in the left and right columns. Similar language organizers could be developed for other topics.

A Frayer model is a specific graphic organizer that is useful when vocabulary terms are confusing or closely related (Barton and Heidema 2002). The model contains four sections: definition, facts, examples, and nonexamples (see **fig. 3** for an example related to the term *variable*). Both research (Adams 2003; Paulsen 2007) and personal experience demonstrate that nonexamples can be particularly powerful in helping refine and clarify definitions. When students ask, "How about this?" or "How about that?" they can refer to the example and nonexample sections. New misunderstandings that are uncovered

can be added to the "nonexample" section. Sometimes substituting sections to suit the situation can be useful—for instance, using essential characteristics and nonessential characteristics or symbolic representation and graphical representation as sections. Students frequently refer to their organizers during lessons or when reviewing for tests.

Provide Repetition and Support Long-Term Retention

All students benefit from repeated exposure to vocabulary; however, ELLs require more repetition to integrate vocabulary into their mathematical understanding. In addition, students may need assistance in organizing their vocabulary knowledge into long-term memory (Adams 2003). Using vocabulary words within context while referring to the definitions (Echevarria, Vogt, and Short 2010) can be helpful. Providing different examples or diagrams each time the word is used helps avoid confusion and brings depth to students' growing understanding.

Word Walls

Reinforcing vocabulary from the interactive word wall can support long-term retention. A simple idea is to take four to five minutes at the end of class to play password or charades, using words from current or previous word walls. Another idea is to encourage and facilitate instructional conversations (Cazden 2001) that can support long-term retention of mathematics language and build meaning about mathematical concepts (NCTM 2000). Word walls can scaffold these conversations. When small groups discuss a mathematics problem, points can be awarded for appropriate use of words from the word wall—for example, using words such as *formula*, *variables*, *equations*, *graphs,* and *order of operations* when discussing using algebra in the real world.

Graphic Organizers

The graphic organizers used throughout a unit can and should be revisited to support long-term retention. In addition, new graphic organizers can be introduced to review previously learned vocabulary and concepts. For example, an organizer with a formal definition, specific properties or special cases, and some examples could be used to review the concept of factors (see **fig. 4**).

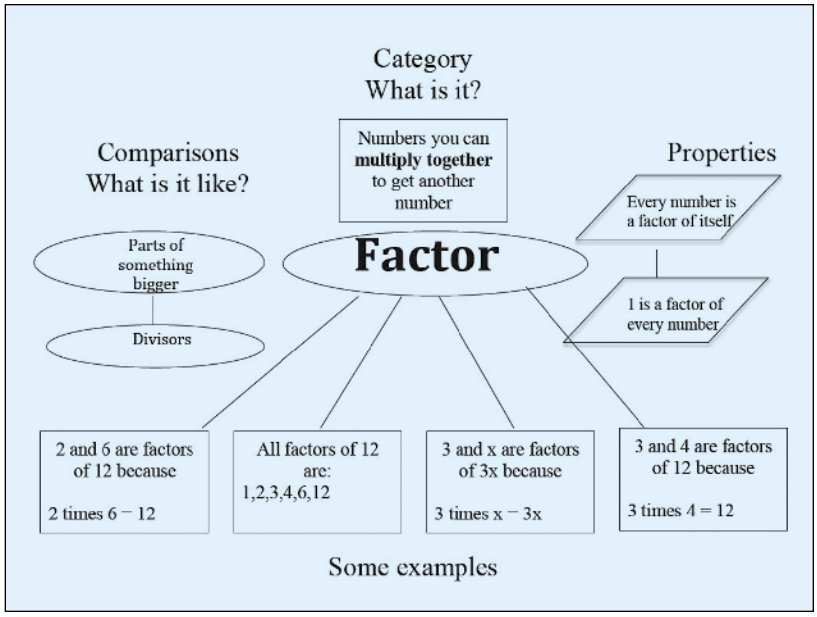

Fig. 4 This graphic organizer would be useful during review of the concept of factors.

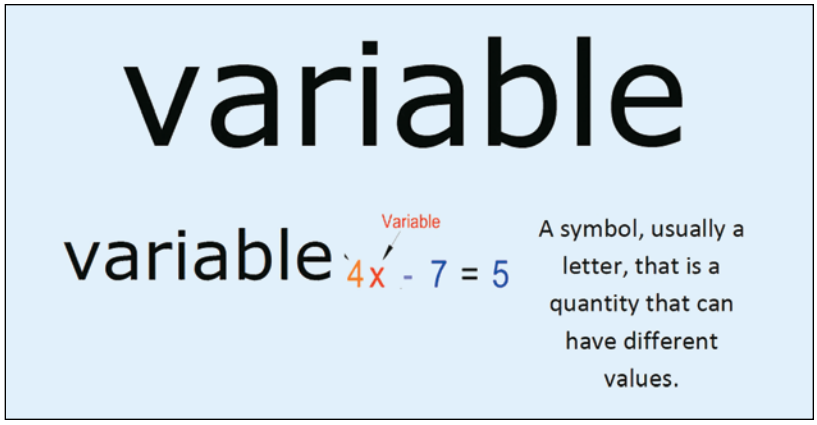

Fig. 5 This word wall entry can be folded so that only the vocabulary word is showing.

TEACHER AWARENESS

Along with reading research literature, mathematics teachers should build their own understanding of the challenges that their ELL students face. Awareness of the confusion caused by symbols and diagrams, concepts that can be represented with multiple terms, words that have multiple meanings, and the overlap between mathematics vocabulary and everyday usage can help teachers provide appropriate emphasis or explicit teaching.

HELPFUL HINTS

Word Walls

A simple way to make a word wall is to use a hanging pocket schedule organizer (typically used by elementary school teachers). After deciding on the unit vocabulary list (see **table 3**), type the words into a document (in landscape mode), with one word on each line. Center each word and enlarge it so that it fills a line of the paper. On the next line, type the word, its definition, a diagram, and an example. After printing, fold the paper so that the word is on one side and the expanded definition is on the other (see **fig. 5**). Slide the pieces into the organizer with the words showing. As the unit progresses and the words are discussed in context, reverse the paper so that the expanded definition is revealed.

*Graphic Organizer*s

Many Internet sites—for example, CAST (www.cast.org) and Thinkport (www.thinkport.org)—have sample graphic organizers that can be used as is or customized. Teachers need not limit themselves to mathematics organizers; many excellent vocabulary organizers, such as Frayer models, come from other content areas. Providing a graphic organizer can help connect content within the unit and then can be used later as a review. Colored paper can assist with organization. In my class, colored paper means "keep it forever." Color makes important graphic organizers easy to find (I can say, "Pull out the red graphic organizer on variables"). At the end of the year, unit organizers make a good, concise way to review.

REFLECTIONS AND RECOMMENDATIONS

As I reflect on my experiences and those of my students, I am reminded of Jorge's confusion about mathematics vocabulary. His question has led me to increase my own awareness of the challenges related to mathematics vocabulary that ELL students face and strategies that I might use to support these students.

To help ELL students develop essential mathematical practices (CCSSI 2010), I recommend the use of word walls and graphic organizers to support vocabulary development. Specifically, I recommend the following:

- Select vocabulary words for a unit and post these on the day that the unit is introduced.
- Assess students' current understanding.
- Refer to the words throughout the unit, adding to the definitions and giving context.
- Provide frequent opportunities for students' misunderstanding to come to light.
- Use graphic organizers to help clarify the meaning of words and support long-term retention of vocabulary.

In addition to using word walls and graphic organizers, teachers should continue to investigate ideas available through books, journal articles, and websites (there are lots of good ideas out there). And, of course, listen to your students—that's the first step in supporting them.

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