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Mapping Mathematics Classroom Discourse and Its Implications for Models of Teaching

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This article reports on models of teaching that developed as outgrowths of a study of middle-grades mathematics classes. Grounded theory methodology and sociolinguistic tools were used to move from classroom observations and interviews to line-by-line coding of classroom discourse, to mapping the flow of talk and verbal assessment moves, to a multilevel analysis of the relationships of forms of talk and verbal assessment, and, ultimately, to models of teaching that promote discourse on a continuum from univocal (conveying meaning) to dialogic (constructing meaning through dialogue). Three specific cases are highlighted that represent *deductive* (associated with univocal), *inductive* (associated with dialogic), and *mixed* (a hybrid of deductive and inductive) models of teaching. Teaching practices associated with each model are illustrated and discussed.

Key words: Communication; Discourse Analysis; Grounded theory; Language and mathematics; Metacognition; Middle grades, 5–8; Teachers (role, style, methods); Teaching practice

Over the past 2 decades, reform efforts have identified communication as essential to the teaching and learning of mathematics (NCTM, 1989, 1991, 2000). Although meaningful discourse can enhance learning, the mere presence of talk does not ensure that understanding follows. The *quality* and *type* of discourse are crucial to helping students think conceptually about mathematics (Kazemi & Stipek, 2001; Lampert & Blunk, 1998; Nathan & Knuth, 2003; van Oers, 2002; Van Zoest & Enyart, 1998). When the intention of discourse is to produce "a maximally accurate transmission of a message" (Lotman, 1988, p. 68), it is referred to as *univocal*. In contrast, when discourse is characterized by give-and-take communication that uses dialogue as a process for thinking, it is characterized as *dialogic* (Bakhtin, 1930s/1981; Knuth & Peressini, 2001; Wertsch, 1998; Wood, 1998). There is evidence to suggest that conceptual understanding is more likely to be associated with dialogic discourse than with univocal discourse (Knuth & Peressini, 2001;

The research reported in this article is based on a portion of the first author's doctoral dissertation study at the University of Connecticut under the direction of the second author. See Truxaw and DeFranco (2007) for a related article that focuses on one of the teachers and on a simpler interpretation of the inductive model. The authors gratefully acknowledge valuable comments from anonymous reviewers.

Wertsch & Toma, 1995; Wood, 1998); yet, there are those who argue that "telling" should not be eliminated from teachers' repertoires (Lobato, Clarke, & Ellis, 2005).

Numerous studies have focused on classroom discourse (e.g., Barnes, 1992; Cazden, 2001), its structures (e.g., Coulthard & Brazil, 1981; Mehan, 1985), and its functions (e.g., Brendefur & Frykholm, 2000; Wells, 1999). Further, discourse specifically related to mathematics instruction has been explored (e.g., Bartolini Bussi, 1998; Cobb, Yackel, & McClain, 2000; Hufferd-Ackles, Fuson, & Sherin, 2004; Lampert & Blunk, 1998; Sherin, 2002; Stigler, Fernandez, & Yoshida, 1996), including attention to dialogic and univocal functions (e.g., Nystrand, Wu, Gamoran, Zeiser, & Long, 2003; Peressini & Knuth, 1998; Werstch & Toma, 1995; Wood, 1998). Because current research supports the conviction that effective teaching is a significant, if not the most significant indicator of student achievement and success (Darling-Hammond, 2000; Sanders, 1998), this study sought to examine the teacher's role in the orchestration of meaningful discourse. The results of investigating and describing teaching models illustrative of univocal and dialogic discourse could help to inform appropriate pedagogical choices. Therefore, this article describes and discusses the development and implications of three models of teaching that promote verbal discourse on a continuum from univocal to dialogic.

BACKGROUND

Sociocultural theory, with its contention that higher mental functions derive from social interaction, provides the primary framework for analysis and discussion of discourse as a mediating tool in the teaching-learning process. Specifically, verbal interactions can help to develop back and forth processes from thought to word and from word to thought that allow learners to move beyond what might be easy for them to grasp on their own (Forman, Minick, & Stone, 1993; Vygotsky, 1978, 1934/2002; Wells, 1999; Wertsch, 1985, 1991, 1998). In addition, theories of language (e.g., linguistics, sociolinguistics,[1] and semiotics[2]) provide complementary ideas relevant to understanding the role of discourse in sense-making. Although many linguists have viewed language primarily as a transmitting device (e.g., de Saussure, 1959; Lotman, 1990/2000), some theorists have shifted viewpoints to align with sociocultural perspectives (e.g., Cazden, 1993; Wertsch, 1991), acknowledging interrelationships between thought and speech. Bakhtin (1979/1986) proposed an ecological approach to language that recognizes complex links in utterances that are "filled with dialogic overtones" (p. 92).

When considering language as a mediator of meaning, it is useful to take into account the two main intentions of communication: "to produce a maximally accurate transmission of a message" and "to create a new message in the course of the transmission" (Lotman, 1990/2000, p. 68), characterized as univocal and dialogic

[1]Sociolinguistics are aspects of linguistics that are applied toward connections between language and society (Halliday, 1978; Sinclair & Coulthard, 1975).

[2]Semiotics is the study of all systems of signs and symbols and how they are used to communicate meanings (Lemke, 1990).

discourse, respectively (Wertsch, 1998). Univocal discourse could be imagined with a conduit metaphor, with knowledge being sent in one direction. In contrast, dialogic discourse involves dialogue between at least two voices where some form of transformation takes place and new meaning is generated. Univocal discourse may serve to establish common meaning (Lotman, 1988), whereas dialogic discourse "tends toward dynamism, heterogeneity, and conflict among voices" (Wertsch, 1998, p. 115).

Achieving meaningful classroom discourse is a complex matter. Studies show that U.S. teachers tend to use a transmission style of classroom communication, stating information rather than developing ideas with their students, and offering little opportunity for students to justify, explore, or make meaning for themselves (NCES, 1999, 2000, 2001; Stigler & Hiebert, 1998; U.S. Department of Education, 2000). However, recent evidence suggests that simply engaging students in classroom discourse is not a panacea for improving mathematical understanding and achievement. For instance, merely increasing the quantity of student talk may not improve mathematical understanding because the students may not have the resources to construct or verify mathematical ideas or conventions (Nathan & Knuth, 2003). How discourse is mediated, as well as the interactions of the talk, the verbal assessment, and the mathematical content, are vital to developing mathematical meaning. This suggests that meaningful discourse may appropriately include some "telling" as a "system of actions," as long as the teacher focuses attention on the "development of the students' mathematics rather than on the communication of the teacher's mathematics" (Lobato et al., 2005, p. 109). Indeed, experienced teachers exercise professional judgment related to when and how to shift roles, when to "step in" as a participant and when to "step out" to become a commentator of rules, norms, and concepts (Rittenhouse, 1998).

The most commonly identified pattern of classroom discourse follows the three-part exchange of teacher initiation, student response, and teacher evaluation or follow-up (i.e., IRE or IRF) (Cazden, 2001; Coulthard & Brazil, 1981; Mehan, 1985). This triadic structure has been criticized as encouraging "illusory participation"—that is, participation that is "high on quantity, low on quality"—because "it gives the teacher almost total control of classroom dialogue and social interaction" (Lemke, 1990, p. 168). However, Nassaji and Wells (2000) found that triadic dialogue was the dominant structure within inquiry-style instruction as well. Further, it was noted that within triadic exchanges, the teacher's verbal moves influence the function of the discourse. For example, when follow-up moves are used to evaluate a student's response, the intention of the discourse is likely to tend toward transmitting meaning (i.e., univocal). In contrast, questions that invite students to contribute ideas that might change or modify a discussion are more likely to be associated with dialogic discourse (Wells, 1999; Wersch, 1998).

In addition to functions and structures, verbal moves are also important to the flow of classroom discourse. Although verbal moves have been described in a variety of ways in the literature (e.g., Barnes, 1992; Chapin, O'Connor, & Anderson, 2003; Kazemi & Stipek, 2001; Nystrand, 1997; Nystrand et al., 2003; Wood, 1998), this research focuses specifically on the following types of talk and forms of verbal assessment: *monologic talk* (involves one speaker, usually the teacher, with no expectation of verbal response), *leading talk* (when the verbal exchanges have been controlled by the teacher and lead toward the teacher's point of view), *exploratory talk* (speaking without answers fully intact, analogous to preliminary drafts in writing) (Barnes, 1992; Cazden, 2001), *accountable talk* (talk that requires accountability to accurate and appropriate knowledge, to rigorous standards of reasoning, and to the learning community) (Michaels, O'Connor, Hall, & Resnick, 2002), *inert assessment* (IA) (assessment that does not incorporate students' understanding into subsequent moves, but rather, guides instruction by keeping the flow and function relatively constant), and *generative assessment* (GA) (assessment that mediates discourse to promote students' active monitoring and regulation of thinking about the mathematics being taught). In this study, the type of talk typically refers to students' verbal moves; for example, a leading talk move may represent a student's response that has been controlled by the teacher so that the student adopts the teacher's point of view. Verbal assessment, both IA and GA, typically refer to the *teacher's* verbal moves, either initiation or follow-up, that influence the flow and function of the talk.

The purpose of this study was to develop models of teaching consistent with the types of talk, the forms of verbal assessment, and the flow of classroom discourse within selected middle grades mathematics classrooms. To do so, the following research question was addressed: What models of teaching can be developed from middle-grades mathematics classes to illustrate discourse on a continuum from univocal to dialogic?

METHODS AND PROCEDURES

The participants were a purposive sample of seven middle-grades mathematics teachers (grades 4 through 8) who were identified as having characteristics indicative of expertise in teaching mathematics (Darling-Hammond, 2000; Shulman, 2000) as noted by achieving National Board for Professional Teaching Standards (NBPTS) certification in Early Adolescent Mathematics (Bond, Smith, Baker, & Hattie, 2000); being recognized as recipients of the Presidential Award for Excellence in Mathematics and Science Teaching (PAEMST) (Weiss & Raphael, 1996; Weiss, Smith, & Malzahn, 2001); or being recommended by university faculty. This article focuses on three of the seven participants.

Data were collected via semistructured interviews, classroom observations, field notes, audiotapes, and videotapes. Pre- and postobservation interviews included predefined questions designed to uncover background traits related to effective practices, as well as professed knowledge, goals, and beliefs associated with teaching (Schoenfeld, 1999). Additionally, questions and themes that emerged from the classroom observations were discussed and documented. Mathematics lessons were observed, field notes were written, and classroom discourse was audiotaped and videotaped. Audio recordings from interviews and observations were transcribed and coded. In all, 23 interviews (one or more preobservation interviews and one or more postobservation interviews for each *set* of lessons) were conducted with the seven participants and 23 lessons were observed.

Grounded theory methodology (Glaser & Strauss, 1967; Strauss & Corbin, 1990), multiple-case study design (Stake, 1995; Yin, 1994), and sociolinguistic tools (Wells, 1999) were applied within a predominantly sociocultural framework (Vygotsky, 1978, 1934/2002; Wertsch, 1985, 1991, 1998) in order to systematically analyze language as a mediator of meaning. Data collected from each participant were analyzed using constant comparison methods (Strauss & Corbin, 1990) so that each set of data would provide additional evidence to inspect, test, and refine the models being developed. Throughout the process, colleagues with expertise and experience both in mathematics education and in discourse analysis served as peer debriefers to provide trustworthiness and reliability of the coding and analysis of the data. For example, peer debriefers coded passages independently and brought them back for comparison and discussion with the researcher. Inconsistencies in coding were discussed until consensus was achieved. Additionally, peer debriefers worked together with the researcher to develop the components of the teaching models.

MODELS OF TEACHING

This article reports on the results of fine-grained analysis of classroom discourse drawn from three teachers (Mr. Larson, Ms. Reardon, and Mr. Townsend; all names are pseudonyms). Three distinct models of teaching were uncovered as a result of the analytic process: an *inductive model* (associated with dialogic discourse), a *deductive model* (associated with univocal discourse), and *a mixed model* (a hybrid of the other two models). The episodes presented were selected from among those analyzed to represent teaching practices associated with verbal discourse on a continuum from univocal to dialogic. Specific coding and analysis procedures are described for the inductive model; then the other two models of teaching are described briefly.

Analysis Leading to a Model of Teaching Based on Mr. Larson's Classroom Discourse

Classroom Setting

The episode under investigation was observed in an eighth-grade honors algebra class (although all eight graders at the school took algebra) in a suburban middle school in New England. The lesson was taught by Mr. Larson, a mathematics teacher with NBPTS certification and 35 years of teaching experience. Students' desks were arranged in groups of four with pairs facing each other. Nineteen students (14 boys and 5 girls) were in attendance.

Coding the Classroom Transcripts

Classroom dialogue transcripts were formatted into tables and numbered based on "utterances" (i.e., speaker's turns; from this point on to be called "lines") (Bakhtin, 1979/1986; Sinclair & Coulthard, 1975). All lines of text were coded using strategies adapted from Wells (1999) and Nassaji and Wells (2000) (see Appendix A for the coding procedures). In particular, analysis included attention to basic structures such as *moves*, *exchanges*, *sequences*, and *episodes* (Mehan, 1985; Sinclair & Coulthard, 1975; Wells, 1999). A *move*, exemplified by a question or an answer from one speaker, is identified as the smallest building block. An *exchange*, made up of two or more moves, occurs between speakers. Typically, the teacher initiates an exchange, the student responds, and the teacher follows up with either an evaluation or some sort of feedback to the student's response. Exchanges are categorized as either *nuclear* (can stand alone) or *bound* (dependent upon or embedded within previous exchanges). A *sequence* is a unit that contains a single nuclear exchange and any exchanges that are bound to it. Finally, an *episode* is composed of all the sequences that are necessary to carry out an activity.

To illustrate how the classroom discourse was coded, lines 4–8 from a coded sequence found in Appendix B are described. The excerpt is from a lesson consisting of 396 lines, 12 sequences, and two episodes. The coding of lines 4–8 indicates that Mr. Larson and his students were engaged in discourse following a triadic exchange structure (IRF) that was being used to establish a definition for prime numbers. Mr. Larson initiated questions, the students responded, and Mr. Larson followed up or initiated new questions (see the Move column). The talk moves were coded as leading talk and exploratory talk (see the Comment column). The verbal assessment moves were coded as inert (IA) (see the Comment column) because they maintained the status quo of the discourse. This line-by-line coding was an important step in the analytic process for two reasons. First, it was used to identify and confirm constructs previously reported in the literature: for example, IRF moves and structure of exchanges. Second, this coding focused on characteristics that were necessary for this investigation in particular: for example, the identification of the specific forms of talk and verbal assessment, as defined in this study.

Developing Sequence Maps

The line-by-line coding helped to uncover relationships between the talk and verbal assessment; however, in order to address larger functions (e.g., univocal versus dialogic), it also was necessary to examine the data on the broader level of the sequence. In fact, although Wells (1999) coded dialogue on the move and exchange level, he noted that the sequence was the "unit which is of greatest functional significance" (p. 236), because sequences involve successive moves and exchanges that are introduced, negotiated, and brought to completion. In order to enhance the analysis at the sequence level, constant comparison methodology (Strauss & Corbin, 1990) was used to develop a graphic model of potential flow of the talk and verbal assessment in mathematics classes (see Appendix C). This graphic model served as a template for creating *sequence maps* (i.e., graphic diagrams representing the flow of forms of talk and verbal assessment within a sequence) from the data.

Specifically, after each line of text was coded, the template of the flow of classroom discourse was used to translate the text to a sequence map. Figure 1 shows a sequence map of the line-by-line coding for sequence 2 in this lesson (see Appendix B). To translate the text to the map, the coded moves within each sequence of a lesson were renumbered beginning with the number 1 and ending with the last move in a particular sequence. For example, in Figure 1 the number 1 represents the first move of sequence 2. Each number on the sequence map represents a verbal move—that is, either a type of talk or a form of verbal assessment. The flow of the discourse can be tracked by following the numbered moves consecutively. A marker X is placed along a line representing a continuum of discourse ranging from univocal to dialogic. The placement along the continuum is not absolute; it acts as a marker indicating *tendencies* of the discourse within a sequence toward univocal or dialogic. To determine univocal and dialogic tendencies, indicators from the research literature were compiled (see Truxaw, 2004); then, the coded transcripts were examined for indicators that informed the *overall purpose* of the discourse within a sequence. The sequence shown in Figure 1 was mapped as tending toward univocal since its *overall purpose* was to convey information (Lotman, 1990/2000). However, since there were some indicators of dialogic discourse (e.g,, GA—moves 11, 15, and 17; exploratory talk—moves 4, 6, 8, and 10; and accountable talk—moves 12, 16, and 18), there was a slight shift along the continuum toward dialogic.

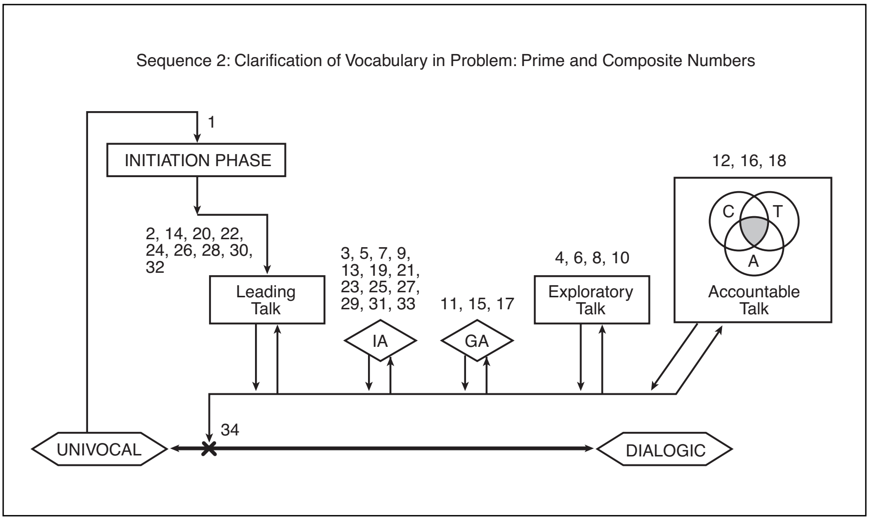


Figure 1. Mr. Larson, map of sequence 2.

Using Sequence Maps to Suggest Further Analysis

Related to this investigation, 120 sequence maps across seven participants were developed and analyzed; this article focuses on 55 sequence maps across three of the seven participants. The sequence maps allowed for visualization of patterns of talk and verbal assessment within individual sequences, across sequences within the same lesson, and across cases. Along with graphically illustrating the flow of talk and verbal assessment, the sequence maps were instrumental in uncovering discursive situations that warranted further analysis:

- *Recurring cycles:* when the talk and verbal assessment moves repeatedly cycled between one form of talk and one form of verbal assessment;
- *Pivotal points:* when the talk and verbal assessment moves repeatedly cycled between a particular form of talk and assessment (e.g., leading talk and IA), but then shifted direction (e.g., to accountable talk and GA);
- *Complex sequence maps:* a sequence map that was particularly complex, that is, it included several forms of talk and verbal assessment, including instances of GA and accountable talk;
- *Strong dialogic or univocal tendencies:* a sequence map that included sufficient indicators to categorize it as strongly toward one end or the other of the univocal-dialogic continuum.

For example, in Mr. Larson's case, the map that was developed for sequence 4 (see Figure 2) suggesting the need for deeper analysis; in particular, it was *the most*

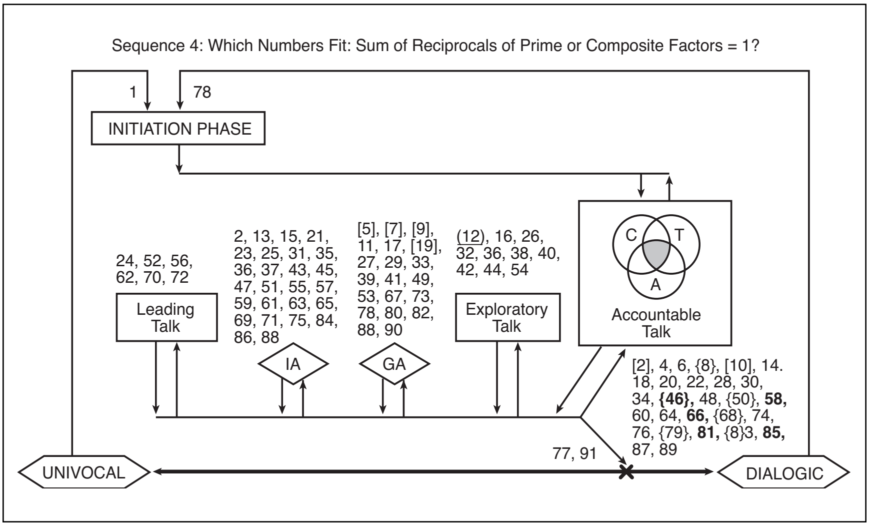


Figure 2. Mr. Larson, map of sequence 4.

complex from among the 120 mapped sequences (across the seven participants' lessons), and as displayed in its sequence map, the discourse tended toward *dialogic*.

Deconstructing the Data

After sequence 4 was identified as one that warranted further investigation, multilevel analysis was undertaken in order to gain a more detailed view of the relationships between the types of talk and the forms of verbal assessment and the functions of the discourse. To begin, the sequence map, the accompanying coded classroom transcripts, the interview transcripts, and the field notes were reexamined. This analysis provided evidence for considering how the content, the flow, and the teacher's intentions might influence the outcomes of the discourse. Next, the text was *deconstructed*—that is, it was divided into subunits according to natural, thematic breaks in the dialogue. Each subunit was represented in multiple ways, including a summary (including evidence from observations and interviews), a graphic subunit map, and text from the transcript (see Figure 3 for an example of subunit analysis).

Reconstructing the Data

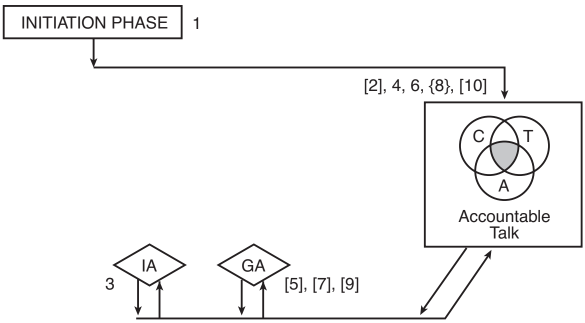
Although deconstruction of sequence 4 achieved a fine-grained view, this analysis also revealed that the preceding three sequences served as a foundation from which sequence 4 was built. Because sequences 1 through 4 together represented an instructional *episode,* this provided a rationale for reexamining the data within the larger, episodic context as well. This analysis involved deconstruction not only of sequence 4, but also of sequences 1 through 3, which resulted in a total of 15 subunits.

After each of the sequences within the episode was deconstructed into subunits, patterns within and across the subunits were identified. In particular, the analysis of this episode showed that components (e.g., working from a frame of reference, establishing shared meaning, investigating, and hypothesizing) *recurred cyclically*. The recursive patterns suggested a model of teaching associated with the episode in Mr. Larson's mathematics class. The subunits were *reconstructed* by mapping them onto the identified components and then graphically representing them. The reconstruction process provided a means for linking the types of verbal moves (i.e., talk and verbal assessment) with the associated components of the teaching model, thus focusing on *when* and *how* specific verbal moves may be productive. The resulting model represents a pattern of cyclical components demonstrating that instruction was predominantly inductive, moving from specific cases, through conjectures based on plausible inferences, toward more general hypotheses and rules (Pólya, 1954, 1985); thus, it was called an *inductive model of teaching* (see Figure 4). The inductive model is associated with discourse that tends toward dialogic.

The efficacy of the model relies not only on the components that were identified during the analysis—in particular, the deconstruction and reconstruction of the

Summary of subunit 4A: In this subunit of sequence 4, Mr. Larson orchestrated testing the outcome of the original problem (that the sum of the reciprocals of the prime and composite factors equals 1) on a different number (i.e., 6). When it worked, Mr. Larson introduced a hypothesis related to the solutions. He named the "hypothesis" after the student who presented the solution to the problem and "wondered" if it would always work. In the interview, Mr. Larson noted that he selected 6 (pretending to select it randomly), because he knew that it would also work, since it was a perfect number.

Subunit 4A map:



Text from transcript:

Mr. Larson: That's sort of surprising that it would actually be 1. I wonder if that's always true. I'm going to try another. I'm going to try 6. Somebody said something about 6. What are the factors of 6? Bruce?

Bruce: Two, 3, 1, and 6.

Mr. Larson: Okay. [Writes factors on board.] So, again, what we said was, we're going to only use the prime and composite factors, right? So we'll throw out this one. [Crosses out the 1.] So we have 1/2 plus 1/3 plus 1/6, right? And I like what B2 did, which was, he made a same denominator. [Shows 3/6 + 2/6 + 1/6 on board.] And what do we get?

Lindsay: 6/6.

Mr. Larson: Which is 1! Whoa! So what should we call this? Should we call this the Hankins Hypothesis or what? You want credit for it, David?

David: Definitely.

Figure 3. Subunit analysis 4A, representing the first of 12 subunits developed from Mr. Larson's sequence 4.

subunits—but also on the connections of these components to when and how specific types of talk and verbal assessment were productively used. Therefore, the descriptions of the components of the model that follow include attention to the patterns of talk and verbal assessment in order to illustrate how *this teacher* orchestrated discourse with overall outcomes that tended toward dialogic.

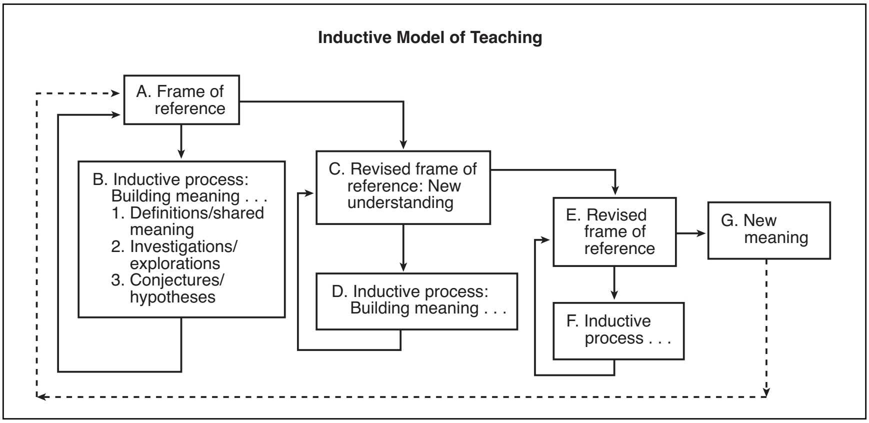


Figure 4. Inductive model of teaching built from an episode in Mr. Larson's mathematics class.

Classroom Discourse Leading to an Inductive Model of Teaching

Establishing a Frame of Reference

In sequence 1, Mr. Larson introduced the following problem to his class: "What is the sum of the reciprocals of the prime or composite factors of 28?" Mr. Larson indicated that the purpose of the episode was to guide his students toward discovering properties surrounding the problem (postlesson interview). He said that the problem had derived from a similar one given in a mathematics competition the previous week—that is, *find the sum of the reciprocals of all the factors of 28* (i.e., 1/1 + 1/2 + 1/4 + 1/7 + 1/14 + 1/28 = 56/28 = 2). Mr. Larson explained his thinking about how the problem led to a learning episode in his mathematics class:

> I particularly liked the first problem and I was surprised at the result, [and] said, "Wow! If you take one off of here, you get one. Is that always going to be true—the reciprocals of the factors?" And I happened to choose six as the next number I tried and it worked again. . . . I said, [inaudible] . . . we've got some[thing] funny . . . it's a perfect number . . . then I said, oh, yeah, 28's a perfect number too. What if it's not a perfect number? I tried another number and it didn't work. And I realized, you know, it only works for perfect numbers. And I thought about how it made sense. And wouldn't it be neat if kids could come to that . . . themselves? So . . . I tried to massage the problem to make it to be that . . . and thought it might lead to a nice discussion about it. (Postlesson interview)

Analysis revealed that the problem introduced in this sequence served as a *frame of reference* from which the rest of the episode was built. This represented the first box within the inductive model (see the A. box in Figure 4). Note that the sequence map associated with this component (see Figure 5) shows the use of monologic talk, leading talk, and IA with an overall outcome of univocal discourse (i.e., conveying meaning).

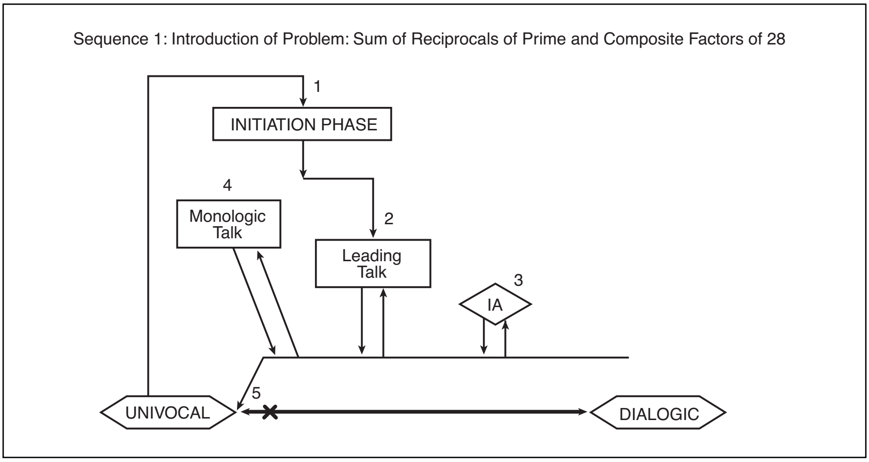


Figure 5. Mr. Larson, map of sequence 1.

Inductive Process: Building Meaning

Subunit analysis of sequences 2 and 3 revealed two components: *establishing definitions and shared meanings* and *investigations and explorations* (see B. in Figure 4). Explanations and examples of these components follow.

Definitions and shared meaning. In sequence 2, Mr. Larson facilitated discussion to clarify the problem and to establish shared understanding of the vocabulary necessary for the problem (the vocabulary of prime and composite numbers). Mr. Larson encouraged students to express their own understanding, using both IA and GA to promote an accurate understanding of the definitions. For example, when a student offered a definition for prime numbers as "Numbers that can only be divided by one and itself," Mr. Larson facilitated an exchange of ideas using examples and counterexamples to illustrate that the number 1 is neither prime nor composite. As a result, a student restated the definition to include "It has exactly two factors." Ensuing dialogue helped the students to agree on a more precise definition of prime numbers. Similar discussion was used to reach consensus on a definition of composite numbers. In sum, this discourse was used to establish *definitions and shared meaning* about the problem. The sequence map associated with this component (see Figure 1) shows the use of leading talk, exploratory talk, accountable talk, IA, and GA, with an overall outcome that tended toward univocal discourse, with a slight shift toward dialogic.

Investigations and explorations. In sequence 3, Mr. Larson asked the students to work in small groups to investigate the problem. Mr. Larson listened and observed, identifying students who might need assistance and also students whom he might call on during large group discussion to provoke meaningful discourse (postlesson interview). When the whole group reconvened, a student volunteered to share his solution:

David: First, I wrote out all the factors I knew of 28. [Lists 1, 2, 4, 7, 14, 28.]

Mr. Larson: Yes. This makes complete sense, to start with a list of all the possible factors.

David: Then I figured out the prime ones. Two and 7 are the prime factors. And 4, 14, and 28 are the composite ones. [Circles numbers as he says them.] So I turned those into their reciprocals [shows reciprocals: 1/2, 1/4, 1/7, 1/14, 1/28]. And then for easier adding, I just flipped them all into 28ths. [Shows equivalent fractions with denominators of 28: 14/28, 7/28, 4/28, 2/ 28, 1/28]

Mr. Larson: Yeah.

David: [David adds the fractions on the white board.] And then when I added them up I got 28/28, which is 1.

After the student completed his explanation, the class, by consensus, agreed that the sum of the reciprocals of the prime and composite factors of 28 equals 1. The investigation and presentation of a solution represented the *investigations and explorations* component of the model of teaching (see B. 2. in Figure 4). Note that the sequence map associated with this component (see Figure 6) shows the use of leading talk, exploratory talk, accountable talk, and IA with an overall outcome that tended toward univocal function (with a slight shift toward dialogic).

The discourse in the first three sequences focused on conveying shared understanding of the problem, investigating it, and finding an answer to it; that is, the discourse was predominantly univocal. It is worth noting that at this juncture, the components developed from the first three sequences established shared meaning and promoted associated investigations and explorations but did not stop there.

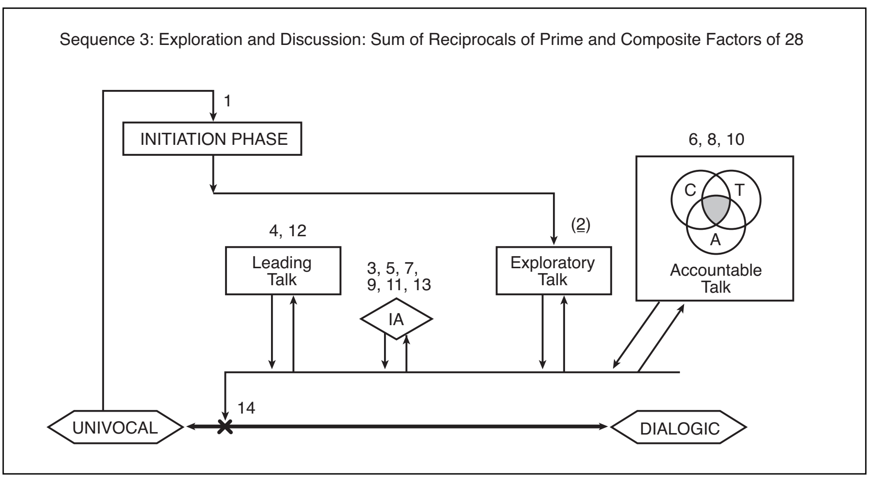


Figure 6. Mr. Larson, map of sequence 3.

Rather, sequences 1, 2, and 3 served as groundwork for the rest of the episode. As will be described, during sequence 4 the nature of the discourse shifted to include more frequent examples of exploratory talk, accountable talk, and GA—and stronger dialogic tendencies.

Conjectures and hypotheses. Analysis of the 12 subunits from sequence 4 revealed that instead of merely accepting the solution of the problem—that is, *the sum of the reciprocals of the prime or composite factors of 28 equals 1*—metacognitive-type assessments (i.e., GA) (Flavell, 1976, 1979) were used to press toward reflection about the problem that moved students toward *conjectures and hypotheses* (see B. 3. in Figure 4). Although Mr. Larson knew the answers to his questions from investigating the problem in advance, instead of using this knowledge to lead the students to specific answers, he used metacognitive prompts to help his students construct or discover the principles involved. For example, after the answer to the initial problem had been presented, Mr. Larson stated, "That's sort of surprising that it would actually be 1. I wonder if that's always true?" He then suggested trying a different number, 6. At this point, classroom discussion was used to demonstrate that the sum of the reciprocals of the prime or composite factors of 6 equals 1. Next, Mr. Larson orchestrated the introduction of the "Hankins Hypothesis," which was named after the boy in class who had presented the solution to the original problem. Mr. Larson wrote the hypothesis on the board: *The sum of reciprocals of the prime and composite factors of a number will always be 1.*[3] This part of sequence 4 mapped onto the *conjectures and hypotheses* component of the model of teaching. The verbal moves associated with this component were predominantly accountable talk and GA.

Revised Frame of Reference

The next box in the model (see C. in Figure 4) was developed after the solution and the hypothesis were infused within the cycle and the original frame of reference was revisited (i.e., "What is the sum of the reciprocals of the prime or composite factors of 28?"). This moved toward a *revised frame of reference* that considered not only the original problem but also additional *shared meaning* that had been discussed; that is, the "Hankins Hypothesis." The strategic use of GA (and, in particular, modeling metacognitive processes), supported the cyclic nature of the discourse.

Recurring Cycles

The cycle continued recursively as *inductive processes* (see D. in Figure 4) were applied to the revised frame of reference. In subunit 4B, Mr. Larson asked students to work in small groups to test the hypothesis in order to further *explore the problem*. As before, knowing the answer to his question, Mr. Larson did not lead students to

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[3]Although Mr. Larson knew that only perfect numbers would yield a result of 1, the "hypothesis" provided a vehicle for further investigation related to number properties (postlesson interview).

his answer but instead used GA to help students monitor their own thinking.

Mr. Larson: We've seen two examples now where it works. I'm sort of surprised . . . I don't know why it would work, but it seems to work. . . . Would you guys check it out? Would you each take some other number and check it to see if, in fact, it does work?

While the students investigated the hypothesis, Mr. Larson circulated around the room, listening and asking questions. For example, when a student said, "It doesn't work for primes," Mr. Larson asked him to think about why that might be so.

Mr. Larson called the class's attention back to the whole group by asking, "Okay, so what did you discover?" When students shared the results of their investigations, Mr. Larson used both IA and GA to facilitate talk that moved from exploratory toward accountable. For example, when students reported that specific numbers did not work, instead of abandoning the hypothesis, Mr. Larson suggested that these cases might be exceptions to the Hankins Hypothesis. Exceptions such as *the hypothesis doesn't work for primes* and *the hypothesis doesn't work for perfect cubes* were named after the students who offered them and were documented on the board. This encouraged the students to look for properties associated with the numbers that worked or did not work for the Hankins Hypothesis.

Analysis of subunits 4C–4F suggested further *conjectures* about the hypothesis, *revisions to the frame of reference* (see E. in Figure 4), and continued *inductive processes* (see F. in Figure 4). The verbal exchanges continued until a student said, "It didn't work for 36, which is an *abundant number*."

Mr. Larson: Whoa! A what? [Dramatically]

David: An abundant number.

Mr. Larson: An *abundant* number! What is an abundant number?

David: When the factors of the number add up to more than the number itself.

Here Mr. Larson encouraged the student to act as the *primary knower,*[4] thus facilitating accountable talk. Discussion followed (as revealed in subunits 4G–4H) that clarified the definition of an *abundant number* and, further, led to the introduction of and discussion about *deficient numbers* and *perfect numbers*. The verbal moves here were similar to those used when the definitions of prime and composite numbers were established earlier; that is, a mix of leading talk, exploratory talk, accountable talk, IA, and GA.

New Meaning

In subunits 4I–4L, Mr. Larson used both IA and GA to encourage the students to verbalize connections between numerical concepts (e.g., abundant, deficient, and perfect numbers), the Hankins Hypothesis, and the original frame of reference. For example, one student, Kohei, realized that the Hankins Hypothesis was not true for *deficient numbers*. Additionally, students were able to identify that the two numbers that worked for the original problem (i.e., 28 and 6) were *perfect numbers*.

[4]The Primary Knower is the person who knows the information and imparts it (Berry, 1981).

Mr. Larson: Perfect number. Well, anybody know any perfect numbers? Daniel?
Daniel: Six.
Mr. Larson: Six is a perfect number. Huh! . . . and, Arthur?
Arthur: Twenty-eight.
Mr. Larson: Twenty-eight is a perfect number. Hmmmmm.

Mr. Larson then began the process of "closing the loop" on this problem by having students modify the Hankins Hypothesis to incorporate this new information.

Mr. Larson: David, would you like to modify the Hankins Hypothesis?
David: They have to be perfect numbers, not just any number.
Mr. Larson: Let's see here. . . . So the sum of the factors of prime and composite [Reads from board as he adjusts the Hankins Hypothesis] . . . sum of the reciprocals of prime and composite factors of a *perfect number* will be 1.

The episode concluded with Mr. Larson challenging the students to test the newly revised Hankins Hypothesis by using the next perfect number. The inductive processes brought about by the verbal interactions resulted in *new meaning* (see G. in Figure 4) being voiced by the students in the form of accountable talk. The cyclical nature of the discourse, combined with the strategic use of verbal moves, allowed the students to share ideas; investigate a problem; make, test, and revise hypotheses; and build new meaning that moved from specific concrete cases toward a more general understanding of mathematical ideas.

Classroom Discourse Leading to a Deductive Model of Teaching

A second model of teaching was generated through analysis of data from Ms. Reardon's classroom discourse. Methods similar to those described for the development of the *inductive model* were employed to build a *deductive model of teaching,* which is a model associated with univocal discourse (see Figure 7). Because the analytic procedures were described in the context of the inductive model, they will not be discussed in detail here. Instead, the explanations will focus on the components of the deductive model and their connections to the classroom discourse.

The deductive model was derived from a learning episode that took place in a seventh-grade mathematics class in an urban middle school in New England. The lesson was taught by Ms. Reardon, a PAEMST recipient with 28 years of teaching experience. Students' desks were arranged in rows—six rows of five desks each. The 21 students in the class (13 boys and 8 girls) were considered to be of "higher-level" mathematics ability (prelesson interview). The overall lesson focused on reviewing for a "celebration of knowledge" (i.e., a formal assessment). "I knew what I had written up on that celebration [i.e., test]—the information I wanted to know they understood and find out what they didn't understand, which is what our whole class was about" (postlesson interview). Ms. Reardon stated that the purpose of the particular *episode* was to review concepts and procedures related to simplifying fractions.

The entire lesson included 537 lines of text and was parsed into 18 sequences and seven episodes. This sequence stood out from among the sequences in this lesson,

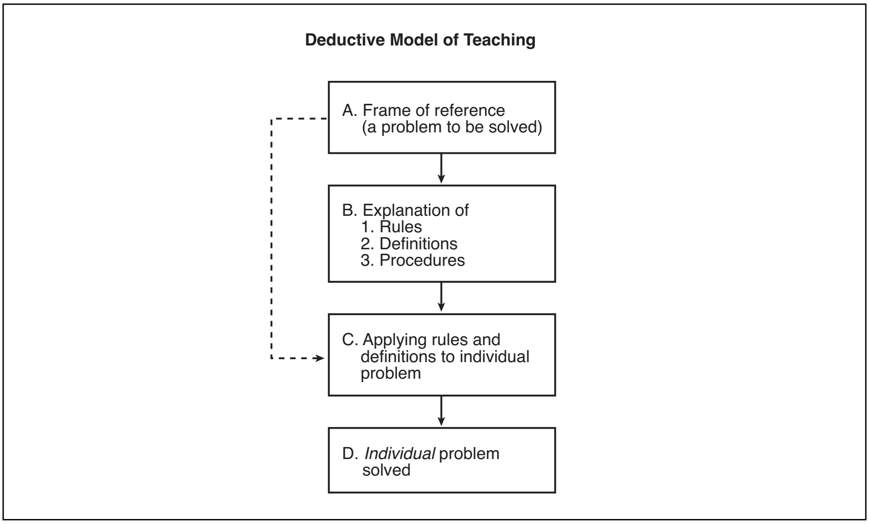

Figure 7. Deductive model of teaching built from an episode in Ms. Reardon's mathematics class.

as well as the 120 mapped sequences from the seven participants. In particular, Ms. Reardon's sequence 5 was selected for multilevel analysis because it included exclusively leading talk and IA with the flow of discourse cycling between them (see Figure 8). Additionally, sequence 5 was one of the longer sequences within the lesson (76 moves), providing an opportunity for deconstruction into subunits, and it was clearly univocal; that is, the function was to convey meaning. As sequence 5 was analyzed, it became clear that it was not connected to other sequences: Sequence 5 served as a self-contained episode where the topics addressed were initiated and brought to closure within one sequence. In this case, the episode consisted of only one sequence. This sequence was decomposed into seven subunits. Using similar analytic procedures to those described for the inductive model, components were identified and the subunits were mapped onto the components of this model of teaching. Specifically, the multilevel analysis revealed that Ms. Reardon used a problem as a frame of reference in order to lead the students through explanations and then applications of rules, definitions, and procedures related to simplifying fractions. The process was deductive in nature—applying general rules to a specific problem in order to work toward an answer.

Establishing a Frame of Reference

Ms. Reardon introduced a problem and clarified its meaning—specifically, the students were asked to simplify the fraction 12/21. This problem provided a *frame of reference* from which the rest of the episode was built (see A. in Figure 7). Leading

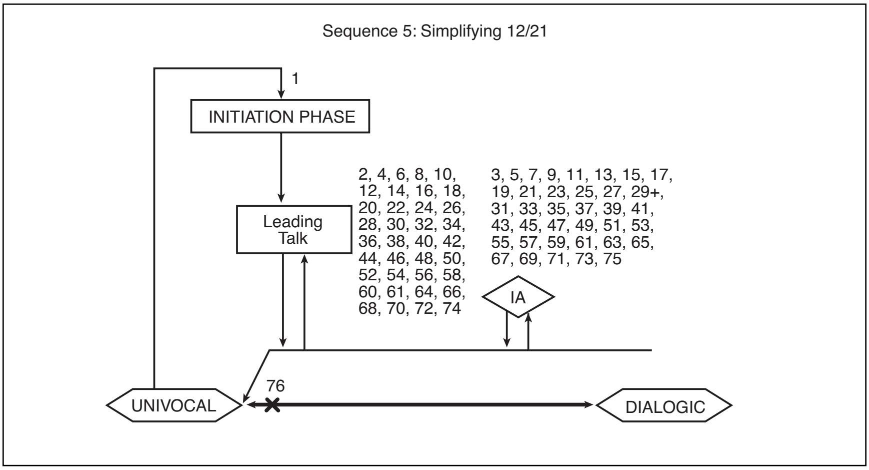

Figure 8. Ms. Reardon, map of sequence 5.

talk and IA exhibiting univocal discourse were associated with this component.

Explanation of Rules, Definitions, and Procedures

Next, Ms. Reardon led the students through an *explanation of rules, definitions, and procedures* related to simplifying the fraction 12/21 (see B. in Figure 7). First she asked the students to list the factors of 12 and then list the factors of 21. The verbal exchanges employed a triadic structure and consisted of leading talk and IA, as illustrated in the following example.

Ms. Reardon: Can you guys list the factors of 12 for me? [Writes on board as she speaks.] Factors of 12. Give me one pair. Lucas.

Lucas: One and 12.

Ms. Reardon: One and 12. And I like to list them as pairs. I find it easier, so I don't leave anything out. [Lists on board]

Sheila: Six and 2.

Ms. Reardon: Six and 2. [Lists on board]

Roberto: Three and 4.

Ms. Reardon: [Lists 3, 4 on board] Any others? [Pauses for 5 seconds] Do you guys agree with this?

Class: Yeah.

The discourse continued to be univocal.

Applying Rules and Definitions to the Individual Problem

Next, Ms. Reardon led the students to finding the common factors of 12 and 21, then their greatest common factor. These verbal interactions were used to *apply rules, definitions, and procedures* to the individual problem (see C. in Figure 7).

This component was associated with leading talk and IA; the discourse continued its tendency toward univocal.

Ms. Reardon: Now I want to know . . . common factors . . . hmmm . . . what do I mean by common? Amanda?

Amanda: You see them more than once.

Ms. Reardon: Yes. We have it once here and once here. I'm going to circle and then write it over here. Somebody tell me one number that appears in both lists.

Taylor: One.

Ms. Reardon: Breanna?

Breanna: Three.

Ms. Reardon: [Pauses, circling the common factors] No more? [No response] Good. Okay. Put the extra comma in, in here. Now, I want the greatest common factor [writes on board], sometimes abbreviated GCF. Greatest common factor. Everybody!

Class: Three.

Individual Problem Solved

After this, Ms. Reardon asked the class to divide the numerator and denominator by 3 (the greatest common factor), thus solving the problem (i.e., 12/21 in simplest form equals 4/7) (see D. in Figure 7). Ms. Reardon then connected the initial frame of reference (i.e., the problem) to a real-world context by comparing different names for the same person (i.e., Ms. Reardon versus her first name, Lydia) with equivalent fractions (i.e., different names for the same value). After re-emphasizing this point, Ms. Reardon pointed to the work on the chalkboard and asked, "So how's this? Good?" and the episode ended. Again, leading talk and IA were associated with this component, as was univocal discourse.

The overall method of instruction was deductive, applying general rules to a specific case. Leading talk and IA were used throughout the episode to transmit meaning to students univocally, rather than to have them generate meaning dialogically. Once the problem was solved, the teaching episode ended.

Classroom Discourse Leading to a Mixed Model of Teaching

Further examination of the data from the seven participants revealed a third model of teaching, the *mixed model of teaching*, that lay somewhere between the inductive and deductive models (see Figure 9). As with the deductive model, essential components of the model and connections to classroom discourse will be described briefly.

The mixed model was based on a learning episode consisting of two sequences within a seventh-grade mathematics class in a suburban middle school in New England. The lesson was taught by Mr. Townsend, a mathematics teacher with NBPTS certification and 13 years teaching experience. Students' desks were arranged in groups of four with pairs facing each other. The 15 students (6 boys and 9 girls) were heterogeneously grouped (prelesson interview). The stated

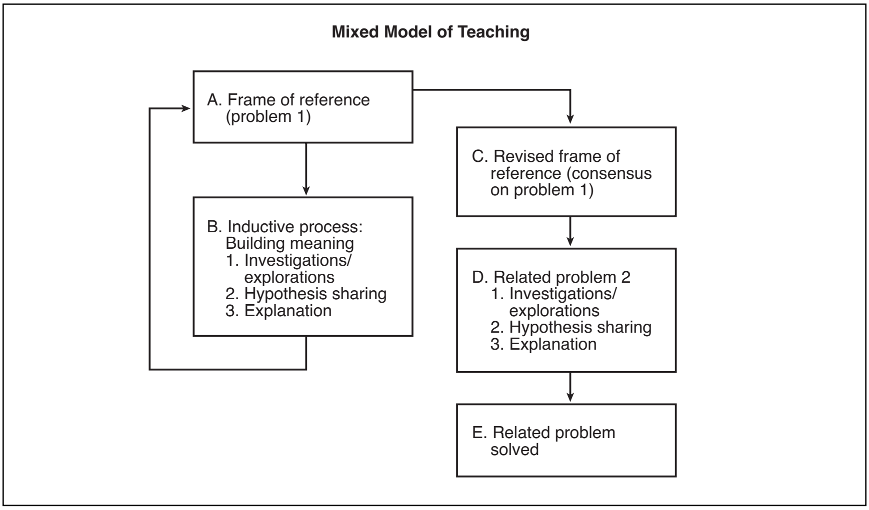

Figure 9. Mixed teaching model built from an episode in Mr. Townsend's mathematics class.

purposes of this episode were to help students make connections between fractions and decimals and to move toward generating an algorithm for multiplying decimals. Mr. Townsend said, "I want to go from where we started thinking about them as fractions problems to starting to look for, to see if they can kind of discover the pattern for basically the algorithm where you're going to multiply them like they're whole numbers" (prelesson interview).

The entire lesson included 599 lines of text and was parsed into 25 sequences and eight episodes. This investigation focused on the third episode in the lesson, which comprised sequences 10 and 11. Sequences 10 and 11 were deconstructed into eight subunits. This episode was targeted for multilevel analysis because analysis of the transcripts and sequence maps (see Figures 10 and 11) revealed mixed indicators. They included a variety of types of talk and both IA and GA (suggesting some tendencies toward dialogic discourse), yet the overall function tended toward univocal. Specifically, the multilevel analysis revealed that Mr. Townsend used two problems as frames of reference and then initiated explorations and sharing of hypotheses. However, in the end, the discourse did not press toward conceptual understanding (Kazemi & Stipek, 2001); instead, the discourse funneled (Wood, 1998) toward specific answers.

Establishing a Frame of Reference

Sequence 10 began with Mr. Townsend introducing a problem to the class—to consider whether 1.25 times 0.5 would be greater than or less than 1—thus establishing a *frame of reference* (see A. in Figure 9). The verbal moves associated with

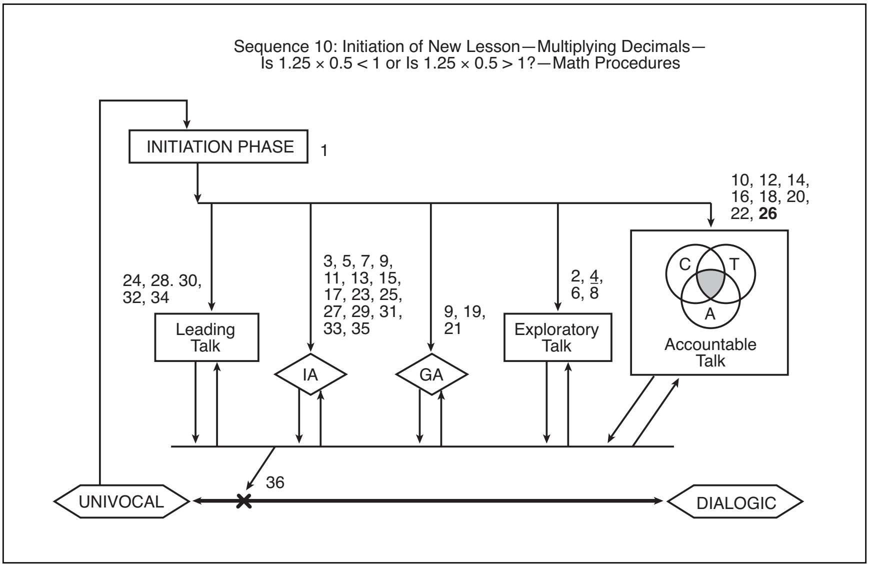

Figure 10. Mr. Townsend, map of sequence 10.

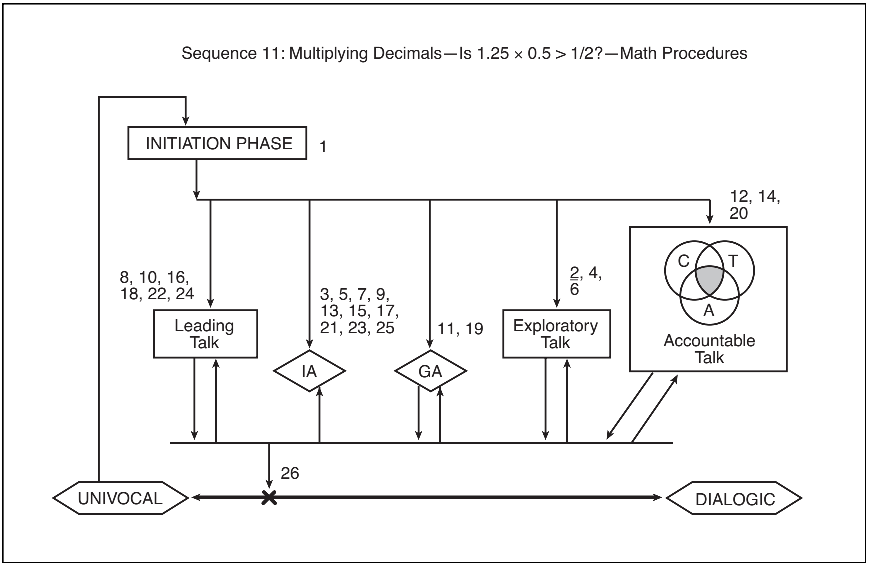

Figure 11. Mr. Townsend, map of sequence 11.

this component included exploratory talk and IA. Although exploratory talk was included, the discourse focused on conveying information about the problem, thus making it univocal.

Inductive Process: Building Meaning

Next, Mr. Townsend asked the students to work in small groups to *explore and discuss conjectures about the problem* (see B. 1. in Figure 9). Again, exploratory talk and IA were associated with this component. He then asked students to *share their hypotheses* (i.e., whether the result would be greater than or less than 1) (see B. 2. in Figure 9). When members of one group reported that they thought that 1.25 times 0.5 would be greater than 1, Mr. Townsend asked them to *explain their thinking* (see B. 3. in Figure 9). The ensuing discussion included examples of exploratory talk, accountable talk, IA, and GA. In the end, the group members opted to revise their answer. Analysis indicated that the small-group conversations and the whole-group discussion were used to *explore, develop, and explain hypotheses* about the frame of reference. Although the use of GA shifted the discourse slightly toward dialogic, overall, it tended toward univocal because it was used to transmit information.

Revised Frame of Reference (Consensus on Problem 1)

Although the students who changed their answer did not generate new meaning substantively, they did revise their conjectures related to the problem. Following this set of verbal exchanges, leading talk and IA were used to move the class to consensus that 1.25 times 0.5 would be less than 1. With this agreement, the *frame of reference* was revisited (see A. in Figure 9) and *revised* to incorporate this new information (see C. in Figure 9). This revised frame of reference appeared to initiate a recursive process similar to the inductive model.

Related Problem 2

Sequence 11 began with Mr. Townsend asking the class to consider a *related problem:* whether 1.25 times 0.5 would be greater than or less than 1/2. Similar to the initial cycle of this teaching model, the students were asked to *investigate* and discuss the related problem in small groups (see D. 1. in Figure 9). Again, exploratory talk and IA were used. The process continued as representatives from the groups were asked to share their *hypotheses* (11-B-2). Leading talk, exploratory talk, and IA were used to move the class to consensus that the result would be greater than one half. The discourse up to this point tended toward univocal, because it focused on conveying procedures and ideas.

At this point, rather than simply accepting the answer, Mr. Townsend infused GA, saying, "Everybody agrees it's greater than a half. Okay. *Convince me*. Why do you think it's going to be greater than a half?" This verbal assessment seemed to be pressing the students toward reasoning and conceptual understanding (Kazemi & Stipek, 2001) related to multiplying fractions and decimals. However, in the ensuing verbal exchanges, a student provided procedural, rather than conceptual, *explanations* associated with the problem (see D. 3. in Figure 9). Again, Mr. Townsend used GA, asking, "So, why does that work out to be greater than a half?" Instead of an explanation or justification, the student responded with what he perceived to be the *answer* to 1.25 times 0.5, saying "I think that gets you 75 hundredths." At this point, Mr. Townsend did not pursue accuracy (1.25 times 0.5 *does not* equal 0.75) nor did he continue to press for reasoning or conceptual understanding. Rather, he shifted the talk to make a connection between the student's answer and money, and the episode ended.

Mr. Townsend: Because if this reminded you of a dollar, dividing by 2 gets you 50 cents?
Ian: Yes.
Mr. Townsend: And then you have a little bit more?
Ian: Yeah.
Mr. Townsend: All right.

In this case, although Mr. Townsend facilitated discussion after an answer to the problem was agreed upon (i.e., $1.25 \times 0.5 > 1/2$), he accepted "low-press" responses (Kazemi & Stipek, 2001) and essentially allowed the discourse to "funnel" (Wood, 1998) toward the answer. When the *related problem was solved*, the learning episode ended (see E. in Figure 9). Although this model had some characteristics similar to the inductive model, it concluded more like the deductive model, with the classroom discourse pointing toward univocal "answers," rather than working toward mathematical meaning.

DISCUSSION

Analysis revealed some clear differences with respect to the types of talk and verbal assessment and the function of the discourse (see Table 1). For example, in comparing the three models, the highest percentages of accountable talk, exploratory talk, and generative assessment were aligned with classroom discourse tending toward dialogic (Mr. Larson—inductive model); the highest percentages of leading talk and inert assessment were aligned with classroom discourse tending toward univocal (Ms. Reardon—deductive model); and percentages of talk and verbal assessment that were somewhere between the other two models led to class-

Table 1
Comparison of Verbal Assessment and Talk Moves Across Episodes

	Verbal assessment		Talk			
	IA	GA	Mono	Lead	Expl	Acct[f]
Mr. Larson (inductive model)	67.7%	33.3%	1.3%	20.8%	32.4%	45.5%
Ms. Reardon (deductive model)	100.0%	0.0%	0.0%	100.0%	0.0%	0.0%
Mr. Townsend (mixed model)	83.3%	16.7%	0.0%	37.9%	24.1%	37.9%

Note. IA = inert assessment; GA = generative assessment; Mono = monologic talk; Lead = leading talk; Expl = exploratory talk; Acct = accountable talk.

room discourse that included tendencies toward both univocal and dialogic (Mr. Townsend—mixed model). The findings that particular verbal moves can be associated with functions and outcomes of discourse are consistent with current research literature (e.g., Barnes, 1992; Chapin et al., 2003; Hufferd-Ackles et al., 2004; Manouchehri & Lapp, 2003; Nystrand, 1997; Wood, 1998). Although the percentages of talk and verbal assessment moves were associated with particular types of discourse along the univocal-dialogic continuum, alone they cannot describe the differences in the classroom discourse associated with each of the models of teaching. In order to do so, a discussion of the models is necessary.

The models did not exist a priori but were built from the data. In particular, associated with each model were components (e.g., within the inductive model, components included a frame of reference, definitions and shared meaning, investigations and explorations, conjectures and hypotheses, etc.). The components were built through the analysis of the data, beginning with the teacher's intentions (i.e., stated aims of instruction); the line-by-line coding of the classroom discourse; the parsing of the coding into sequences, exchanges, and episodes; the representation of each sequence by sequence maps; and, finally, the deconstruction-reconstruction process of the data. Because the models were based on purposefully selected data, they represent, in essence, glimpses of practice rather than "typical" practices for each of these teachers. The research provides rigorously analyzed glimpses of classroom practice that represent discourse on a continuum from univocal to dialogic. Implications will be discussed in the context of the three models.

Inductive Model

As noted earlier, the inductive model of teaching was associated with discourse that tended toward dialogic. To begin, Mr. Larson stated that an instructional goal was to facilitate what he called "guided discovery" (postlesson interview). In particular, Mr. Larson knew what he wanted his students to learn from the episode, a general mathematical principle involving reciprocals of perfect numbers (postlesson interview). He had worked through the problem in advance and anticipated potential pathways for enhancing the learning goals. He was *purposeful and intentional* in using exploratory talk and accountable talk in helping students discover the principle. Further, the discourse moved from relatively univocal, while establishing shared meaning, to relatively dialogic, while constructing new meaning. Once shared meaning was established, Mr. Larson strategically infused metacognitive prompts (i.e., GA) in order to help students actively monitor their own thinking and construct the mathematical principle. Mr. Larson's purposeful involvement aligns with the contention that the teacher's role is vitally important in reform-oriented mathematics instruction (Chazan & Ball, 1995).

In moving students toward an understanding of the mathematical principle dealing with reciprocals of perfect numbers, Mr. Larson opted to use language that helped students to construct or discover the principle, rather than language that led them to the principle. Mr. Larson's classroom discourse (dialogic in nature) was closely aligned with reform-oriented mathematics education, instruction that focuses on students constructing mathematical understanding (NCTM, 2000).

The inductive model offers an approach to teachers who are interested in aligning their teaching with current reform initiatives. Although the model is not meant to be prescriptive, there are some themes and practices that can inform teachers in moving toward using dialogic discourse. For example, exploratory talk, accountable talk, and GA need to be present and used at appropriate times and in appropriate ways. Next, it may be productive to build shared meaning about the mathematics before pressing students toward metacognitive processes that encourage monitoring and regulation of their thinking. Further, the strategic use of metacognitive prompts (GA) seems to be critical in promoting classroom discourse that is dialogic in nature. The cyclical nature of the discourse allows students to revisit and connect ideas, potentially fostering mathematical understanding (Pirie & Kieren, 1989). Finally, the problem chosen for investigation should afford students opportunities to discover new ideas related to the problem, and it should be the teacher's intention that they do so.

Deductive Model

In contrast to the inductive model, the deductive model of teaching was associated with discourse that tended toward univocal. Ms. Reardon's intentions focused on reviewing for a formal assessment. In particular, she was *purposeful and intentional* in her goal of helping students recall how to represent a fraction in simplest form. Throughout the episode, Ms. Reardon used leading talk and IA to direct students to see the rules and procedures for simplifying fractions. In this episode, Ms. Reardon conveyed the rules and procedures, and the discourse ended when students supplied correct answers to her questions.

Clearly, Ms. Reardon used discourse that modeled a more traditional approach to teaching. She felt compelled to cover the material (postlesson interview) and, in contrast to Mr. Larson, she transmitted the information in an efficient way, instead of having students discover the rules for themselves.

The deductive model offers an approach to teachers who are interested in a more didactic, univocal style of teaching. For example, leading talk and IA are associated with conveying information, as are carefully sequenced steps that move from general rules and procedures toward the answer to a particular problem. Inert assessments seem to be instrumental in maintaining a controlled flow and function of the discourse. A cautionary note was offered by Ms. Reardon in a postlesson interview. She asked the researcher, "Do you think that I covered everything in the review that was on [the test]?" The researcher responded, "It looked like it." Ms. Reardon then commented, "One kid out of almost 50 got an A. One. But I *covered* it."

Mixed Model

The mixed model of teaching was associated with discourse that, overall, tended toward univocal but that also included indicators of dialogic discourse. In discussing his intentions for the lesson, Mr. Townsend noted the importance of having students discover algorithms (prelesson interview) but also emphasized computational efficiency (postlesson interview). In comparison to Mr. Larson and Ms. Reardon, Mr. Townsend appeared less *purposeful and intentional* in facilitating the discourse. The episode involved two problems related to multiplying decimals. With each problem, Mr. Townsend initiated an investigation using small-group work and asked group members to report their conjectures. Although Mr. Townsend infused GA, when procedural explanations were offered, he did not "press" for justification (Kazemi & Stipek, 2001); instead, he accepted the responses and moved on. Once conjectures were shared, the talk shifted from exploratory to leading, as Mr. Townsend helped the whole class come to a consensus about the answers to the questions under investigation. Although the episode initially appeared to be opening the conversation up for dialogic discourse, in the end, it was "funneled" (Wood, 1998) toward specific answers, rather than toward building conceptual understanding.

Mr. Townsend's classroom discourse included characteristics of both reform-oriented and more traditional instruction. The use of small-group explorations and students sharing conjectures are aligned with reform initiatives (NCTM, 1991, 2000); computational efficiency, although compatible with the recently released *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006), represents a more traditional stance toward teaching. Within the mixed model, there were components that were similar to those found in both the inductive and deductive models. Although this model seems to offer teachers flexibility in moving along the univocal-dialogic continuum, in the end, the discourse tends to be more univocal in nature.

How the Models Contribute to Our Understanding of Discourse

It is not surprising that these models—reform-oriented, traditional, and something in between—have a familiar feel to them. Although the models appear relatively familiar and simple, the *interplay* of their components with associated talk and verbal assessment provide an interpretation of classroom discourse that moves beyond a surface level. It is not sufficient to advise, "Use these components" or "Use these moves." Rather, the models must be considered in conjunction with associated talk and verbal assessment moves. An additional point is that, although the existence of recurring cycles is characteristic of the inductive model and the lack of these cycles is characteristic of the deductive model, it is not merely the presence or absence of cycles that defines the models. In fact, the models were developed from patterns of verbal moves that were revealed during the multilevel analysis. For example, in the inductive model, the verbal moves provided means for the frame of reference (the original problem) to be revisited cyclically in ways that situated it in a larger context of mathematical concepts, thus moving inductively beyond an answer to the original question toward mathematical meaning-making.

In contrast, within the deductive model, there was an *absence* of verbal moves that revisited the frame of reference on conceptual levels. Although the frame of reference was revisited, it was only to answer the question, not to move beyond it. The analysis of the verbal moves showed that cycles did not exist. In the mixed model, the analysis revealed that some verbal moves were used to revisit the frame of reference; however, these focused more on procedures and less on building conceptual understanding, resulting in limited cycles. Overall, the discourse associated with the mixed model did not move beyond the answers to the questions. With all three models, the coding, analysis, deconstruction, and reconstruction of the data provided opportunities to attend not only to what was said but also to how, when, and why. This multilevel analysis of the data was crucial in developing models that may appear simple but that have important implications about discourse in mathematics classes.

FINAL REMARKS

The three models of teaching presented were based on a fine-grained analysis of the flow and function of discourse within teaching episodes in middle-grades mathematics classes. Although the models represent patterns of discourse that are unique to individual classrooms, they also suggest broader themes and practices associated with discourse that can be described along a continuum from univocal to dialogic. The models present opportunities for different types of discourse on the univocal-dialogic continuum, depending upon the teacher's intentions as well as the appropriate use of talk and verbal assessment moves. It is important to emphasize, however, that the models are not prescriptive nor are they exhaustive. The complexities of when and how the forms of talk and verbal assessment may productively play out in mathematics classrooms need further examination. Further, more research is warranted to further investigate potential models of teaching associated with univocal-dialogic discourse and to explore possible reasons why the discourse and associated teaching and learning progress as they do. The research presented here provides beginnings of strategies and structures from which to build these investigations.

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APPENDIX A: CODING STRATEGIES FOR MOVES

The coding strategies for moves are based on Wells (1999) and Nassaji and Wells (2000); any adaptations or additions made by the authors are underlined.

A	B	C	D	E	F	G	H	I	J
Line#	*Seq#*	Who	Text	K1/ K2	*Exch*	*Move*	Pros	Func	Comment
214	13	T	Good job.	K1	Dep	I	A	Eval+	IA

The lettered columns represent the following:

A. Line # = Line number
B. Seq # = *Sequence* number
C. Who = Speaker: T = teacher; S = student (unidentified); Ss = students
D. Text = Text of verbal discourse

E. K1/K2 = K1= Primary Knower; K2 = Secondary Knower
F. *Exchange* = Type of *exchange*
  1. Nuc = Nuclear—contributes to substantive context
  2. Bound = Bound—depends on nuclear *exchange* ("bound to it")
     a. Dep = Dependent—aspect of nuclear *exchange* developed through specification, exemplification, justification, etc.
     b. Emb = Embedded—deals with problems in the uptake of a *move* in the current exchange, for example, a need for repetition
     c. Prep = Preparatory—further bound category, including acts such as bid-nomination in whole-class question and answer
G. *Move* = Type of *move*
  1. I = Initiation
  2. R = Response
  3. F = Follow-up
H. Pros = Prospectiveness—extent to which *move* determines later *moves*:
  1. D = Demand
  2. G = Give
  3. G+ = Give with added tag making it more strongly prospective
  4. A = Acknowledge
I. Func = Function
  1. Req act = Request action
  2. Req inform = Request information
  3. Req clarif = Request clarification
  4. Req expand = Request expansion/extension of previous contribution
  5. Req examp = Request example
  6. Req sug = Request suggestion
  7. Req opin = Request opinion
  8. Req explan = Request explanation (P = Procedure; C = Concept)
  9. Req justif = Request justification
  10. Req pos/neg = Request yes/no answer

11. Req confirm = Request confirmation
12. Req repeat = Request repetition
13. Req restate = Request restatement of another's contribution
14. Req explore = Request exploration
15. Req ag/dis = Request agree/disagree
16. Req observ = Request observation
17. Req comp = Req comparison/contrast
18. Req apply = Request application of concepts/procedures
19. Act = Action (e.g., getting supplies, correcting work)
20. Check = Check for understanding
21. Chal = Challenge
22. Bid = Request to speak
23. Inf = Give information
24. Sug = Give suggestion
25. Opin = Give opinion
26. Justif = Give justification/explanation
27. Confirm = Give confirmation
28. Qualify = Qualify previous contribution
29. Clarify = Clarify own previous contribution
30. Explain = Explain
31. Extend = Extend/expand previous contribution
32. Examp = Give relevant example
33. Pos/neg = Give yes or no answer
34. Ag/disag = Agree/disagree
35. Observ = State observation
36. Comp = State comparison/contrast
37. Repeat = Repeat own previous contribution
39. Apply = Apply concepts/procedures
40. Nom = Nominate next speaker
41. Acknowl = Acknowledge
42. Accept = Accept previous contribution
43. Reject = Reject previous contribution
44. Eval = Evaluate previous contribution
    a. Eval+ = Positive
    b. Eval– = Negative
45. Reform = Reformulate previous contribution
46. Revise = Revise/change own previous contribution
47. Revoice = Speaker "revoices" another's contribution
48. UpT = Uptake (when one conversant asks someone else about something the other person said previously (Nystrand, 1997).
49. WT = Wait time (when teacher gives students time to think before answering)
50. Sum = Summarize
51. DK = Don't know (speaker expresses that he/she does not know)

J. Comments = Researcher's comments
   1. Univ = *Move* tends toward univocal
   2. Dialog = *Move* tends toward dialogic
   3. Mono = Monologic Talk
   4. Lead = Leading talk
   5. AT = Accountable talk
      a. AT-LC = Accountable to learning community
      b. AT-AAK = Accountable to accurate and appropriate knowledge
         - AT-AAK-F = Fact
         - AT-AAK-P = Procedure
         - AT-AAK-C = Concept
         - AT-AAK-V = Vocabulary
      c. AT-RT = Accountable to rigorous thinking
         - AT-RT-P = Procedure
         - AT-AAK-C = Concept
   6. ET = Exploratory talk
      a. ET-P = Procedure
      b. ET-C = Concept
   7. MT = Metacognitive talk[5]
   8. AQ = Authentic question (not prespecified information)
   9. QAQ = Quasi-authentic questions
   10. TQ = Test question (prespecified, known information)
   11. IA = Inert assessment
      a. IA-P = Procedure
      b. IA-C = Concept
      c. IA-M = Metacognition
   12. GA = Generative Assessment
      a. GA-P = Procedure
      b. GA-C = Concept
      c. GA-M = Metacognition
   13. FR = Fact Response
   14. PE = Procedure explanation
   15. CE = Concept explanation

[5] *Metacognitive talk:* Classroom talk in which participants actively monitor, regulate, and orchestrate their own thinking and learning processes (Flavell, 1976). Although not one of the four identified forms of talk for the study, *metacognitive talk* was identified as potentially *associated with* the other forms of talk. Further, when moves indicated metacognition, they were identified within sequence maps by bracketing the numbered move (e.g., [#] meant that a move included indicators of metacognition).

APPENDIX B: EXAMPLE OF CODING FROM MR. LARSON'S SEQUENCE 2

See Appendix B for explanation of codes. [ ] = noted behaviors/comments; **[inaudible] = inaudible

Map#	Line	Seq	Who	Text	K1 K2	Exch	Mv	Pros	Func	Comment
1	4	2	T	Now I know you all know what a sum is. I think you all know what reciprocals are. Right? What about this business of prime or composite factors? B6?	K1	Nuc	I	D	Inf Req inf Nom	Initiate (Lead) (AT-AAK-V)
2	5	2	B6	**[Inaudible]	K2	Dep	R	G	Inf	Lead[6]
3	6	2	T	Well, let's do the easy one. What do I mean by prime factors of 28? B7?	K1	Dep	I	D	Req inf	IA
4	7	2	B7	Numbers that can only be divided by 1 and itself.	K2	Dep	R	G	Inf	ET (draft)
5	8	2	T	Number that can only be divided by 1 and itself. All right. Is that a prime number?	K1	Dep	F/I	A D	Revoice acknow Req conf	(Not final form) IA
6	9	2	Ss	[Mixed responses]	K2	Dep	R	G	Pos/neg	ET
7	10	2	T	So 7 can only be divided by 1 and 7. So, therefore, 7 is a prime number. Right?	K1	Dep	F/I	G+	Examp Req conf	IA
8	11	2	Ss	Yeah.	K2	Dep	R	G	Conf	ET-incomplete
9	12	2	T	Fourteen can be divided by 1, 2, 7, 14, so it's not a prime number. Agreed?	K1	Dep	F/I	G+ D	Examp Req conf	IA ET (draft)
10	13	2	Ss	Yeah.	K2	Dep	R	G	conf	ET-incomplete
11	14	2	T	But Nathan says, hold on a second because. . . .	K1	Dep	F/I	G+	Req expl	GA-
12	15	2	B4	It has to go out evenly, it can't be a fraction.	K2	Dep	R	G	Chal/expl	ET/AT-AAK
13	16	2	T	Well, when we say divide by, you're right. . . . I mean, when we say divide by, we assume that means that there's no remainder when we divide. Right?	K1	Dep	F/I	A	Eval/ extend req conf	IA Lead (redirects)
14	17		Ss	[Nod]	K2	Dep	R	G	Pos/neg	Lead

[6] Although inaudible, line 5 (Map move 2) was coded as leading talk, because it was a response to a leading question.

Map#	Line	Seq	Who	Text	K1 K2	Exch	Mv	Pros	Func	Comment
15	18	2	T	Everybody comfortable with that definition of a prime? It can only be divided by 1 and itself? Everyone's comfortable except for B10.	K1	Dep	I	D	Req conf Nom	GA AT/C
16	19	2	B10	It has exactly two factors.	K2	Dep	R	G	Inf	AT/AAK
17	20	2	T	It has exactly . . . well, how is that different?	K1	Dep	F/I	A/D	Revoice Req expl	GA
18	21	2	B10	Well, because 1. . .	K2	Dep	R	G	Explain	AT
19	22	2	T	The number 1?	K1	Dep	F	A/D	Revoice	IA/Lead
20	23	2	B10	. . . can divide itself, but it only has one factor.	K2	Dep	R	G	Clarif	Lead completes expected response
21	24	2	T	Ah! Let's consider that special number 1. One is divisible only by 1 and itself, right? It fits B7's definition. But B10 says, eh-eh, not prime because it needs to have two factors. So do people agree with B10?	K1	Dep	F/I	D	Inf/ acknow Req agr/dis	IA considers accuracy (AT- AAK)
22	25	2	Ss	Yeah.	K2	Dep	R	G	Agree	Lead
23	26	2	T	Does everybody agree one is not a prime number? Right?	K1	Dep	F/I	D	Req ag/dis	IA-/(toward lead)
24	27	2	Ss	Yeah.		Dep	R	G	Agree	Lead
25	28	2	T	Because 1 only has one factor. Primes need to have exactly two factors. Right, G2?	K1	Dep	F/I	A D	Revoice Req ag/dis	IA (toward lead) AT-AAK-V
26	29	2	G2	Yes.	K2	Dep	R	G	Agree	Lead
27	30	2	T	Okay. So what are these things called composite numbers? Yes, B5.	K1	Dep	G/I	D	Req inf Nom	IA (toward AT-AAK)
28	31	2	B5	A whole number with more than one prime factor, for example 8, 9, 27, and 51, are composite numbers.	K2	Dep	R	G	Inf	[Appears to be reading this.]
29	32	2	T	Okay, that's a confusing way to say it. Can somebody say it in English? Yes, B4.	K1	Dep	F/I	A D	Req clarify Nom	IA

Map#	Line	Seq	Who	Text	K1 K2	Exch	Mv	Pros	Func	Comment
30	33	2	B4	A whole number with more than two factors.	K2	Dep	R	G	Inf	Lead
31	34	2	T	Yes. A prime number [he meant composite number] is actually, and I would not include the whole number, be because that suggests that 0 is being considered and it's not. So I would just look at the counting numbers. So it's a counting number that has more than two factors. So an example of a composite number, B12, would be…?	K1	Dep	F I	A D	Acknow Reform Req examp	(AT-AAK-V, C) IA
32	35	2	B12	Six.	K2	Dep	R	G	Inf	Lead
33, 34	36	2	T	Six. Sure. Because 1, 2, 3, 6. Four factors, more than two. Composite number. Okay?	K1	Dep	F	A D	Revoice, Extend	IA

APPENDIX C: MODEL OF THE FLOW OF CLASSROOM DISCOURSE

The graphic representation of the model of discourse in Figure C1 shows *possible* components and pathways, not necessarily what will occur in every mathematics lesson. The model serves as a template for creating sequence maps. The basic components of the model include the four forms of talk and the two forms of assessment. Within the model, GA stands for generative assessment; IA stands for inert assessment; and the A, T, and C within the Venn diagram represent the facets of accountable talk (i.e., A = accurate and appropriate knowledge; T = rigorous thinking; and C = community). The lines indicate the flow of discourse. The dashed and dotted lines (that extend from the solid line connecting the forms of talk and assessment) indicate tendencies toward univocal or dialogic discourse. As the discourse progresses, its overall function typically tends more toward either univocal or dialogic. For example, if the discourse were predominantly univocal, it would tend toward the left side of the double-arrowed line. The placement along the double-arrowed line represents a *tendency toward* one or the other, not an absolute position. The placement of the discourse more toward univocal or more toward dialogic is based on indicators (see Truxaw, 2004) that appeared within the coded transcripts from which the discourse had been mapped.

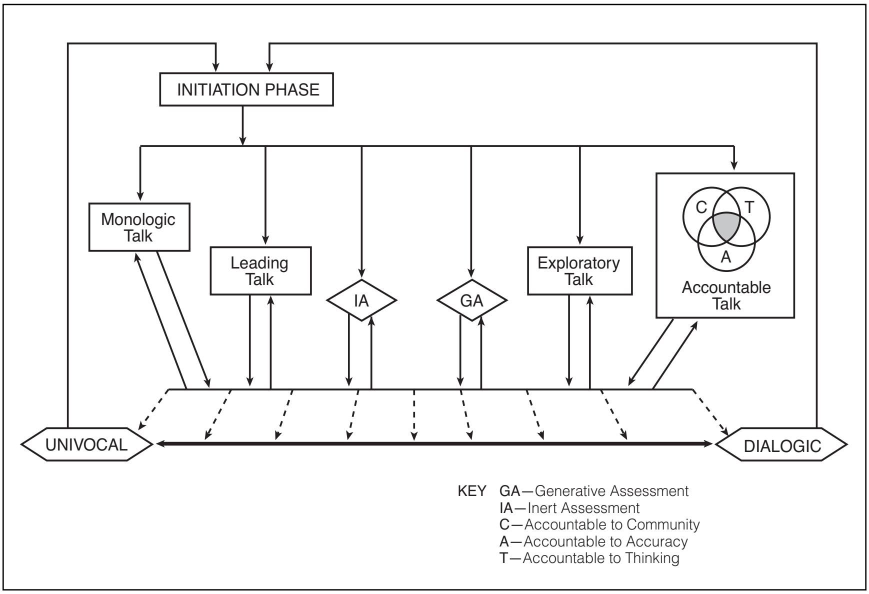

Figure C1. Model of the flow of classroom discourse.