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Teaching for Excellence and Equity in Mathematics

TODOS
Mathematics for ALL

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TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

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IN THIS ISSUE

From the Editors

Let the globe, if nothing else, say this is true:
That even as we grieved, we grew
That even as we hurt, we hoped
That even as we tired, we tried
That we'll forever be tied together, victorious
Not because we will never again know defeat
but because we will never again sow division
….
For there is always light,
if only we're brave enough to see it
If only we're brave enough to be it

(From "The Hill We Climb" by Amanda Gorman, January 20, 2021)

We open this editorial with some powerful verses from Amanda Gorman as a call for reflection and a sign of hope for the year(s) ahead of us. This past year was quite a year, marked not only by the coronavirus pandemic and its devastating impact, especially on Black, Latinx, and Native American communities, but also by the pandemic of systemic racism. True to the core values reflected in its mission, TODOS developed a position statement, "The Mo(ve)ment to Prioritize Antiracist Mathematics: Planning for This and Every School Year" (https://www.todos-math.org/statements). We encourage all our readers to read this position statement and the accompanying commentaries, which provide specific actions that we can all take to prioritize antiracist mathematics teaching and learning.

We are aware how overwhelming the current situation is and that many of us may be juggling many personal and professional responsibilities. Yet, we would like to encourage you to consider submitting an article to *TEEM*. Your voice and experiences are important, as we see how a just, healthy society requires an educated, numerate public. If you have an idea and are not sure if or how it may fit, please contact us (teem@todos-math.org) and we will be happy to discuss it with you. We are very interested in articles written by or with classroom teachers. Your experiences are extremely valuable to all us, even more so in these difficult times that we are all experiencing. Articles on how to facilitate quality equitable mathematics learning in online environments are also welcome. Also, if you have an idea for a special issue, feel free to email us. Keep in mind the guidelines and scope for *TEEM* (https://www.todos-math.org/newteemwb) and the TODOS mission (https://www.todos-math.org/mission-goals).

This issue of *TEEM* has three externally-reviewed articles on different topics, all in the context of mathematics and bilingual learners. In "Growth Mindset to Support Equity and Excellence in Linguistically Diverse Elementary Classrooms," Jessica Guo, Mary Truxaw, and Rebecca Eckert present evidence from an action research project in a first-grade classroom that supports growth mindset and productive struggle of linguistically diverse learners. The authors also provide example of lessons and resources.

In "Engaging Bilingual Mathematics Learners with Principle-Based Instruction," Sarah Roberts illustrates how middle school teachers use four key principles to engage bilingual students in meaningful mathematical work. The principles serve as a guide for other mathematics teachers working with bilingual students in their classes.

In "On the Meaning of Young Children's Mathematics Learning," Cristina Valencia Mazzanti reflects on her experiences in a kindergarten class to question the messages we send to young children about the meanings of mathematics. The author argues for meaningful contexts that center interconnectedness among children and with their communities.

As always, as editors, we are extremely grateful for the dedication and expertise of all our reviewers and authors. We are also very appreciative of the excellent editorial support provided by Associate Editor Lawrence M. Lesser and Layout Editor Susie W. Håkansson. *TEEM* gratefully acknowledges the support of all the leaders in our sponsoring organization, TODOS: Mathematics for ALL. We hope *TEEM* continues to serve the TODOS membership and that this issue serves as a resource for the community and a source of inspiration for future contributions to the journal.

Marta Civil, Anthony Fernandes, Ksenija Simic-Muller, M. Alejandra Sorto, and Craig Willey

Growth Mindset to Support Equity and Excellence in Linguistically Diverse Elementary Classrooms

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Abstract

Research suggests cognitive advantages of speaking more than one language – for example, cognitive flexibility, higher order thinking skills, and better problem solving. We posit that strengths associated with bilingualism suggest potential for a growth mindset and productive struggle that could support mathematical problem solving. This article investigates mathematics teaching and learning practices designed to uncover and support a growth mindset and productive struggle in linguistically diverse elementary classrooms – with an eye toward equity and excellence. We share and discuss lessons learned from educational literature, professional practice, and evidence from a small-scale growth mindset action research project in a linguistically diverse first-grade classroom.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What strengths and challenges do you think emerging bilingual students bring to the classroom that can help or hinder their problem solving in mathematics?
2. What do you know about growth mindset?
3. What are your current experiences or perceptions about how growth mindset, productive struggle, and problem solving do (or could) play out in linguistically diverse mathematics classrooms?
4. What are your thoughts about how emerging bilingual students are positioned to engage in productive struggle and problem solving in linguistically diverse classrooms?

Jessica Guo (jessicaguo@gmail.com) is an elementary school teacher in Los Angeles Unified School District in Los Angeles, CA. She is interested in helping students from diverse backgrounds adapt growth mindsets to transform their outlook on learning, mistakes, and challenges.

Mary Truxaw (mary.truxaw@theteachingcorner.com) recently retired from her position as associate professor in the Neag School of Education at the University of Connecticut in Storrs. Her research interests focus on intersections of mathematics, language, equity, and collaborative teaching practices – especially within the context of linguistically and culturally diverse classrooms.

Rebecca Eckert (rebecca.eckert@uconn.edu) is an associate clinical professor in the Neag School of Education at the University of Connecticut in Storrs. Her research interests include classroom impact of co-teaching strategies, recruitment and preparation of new teachers, arts in the schools, and public policy in promoting equitable learning opportunities for students.

Growth Mindset to Support Equity and Excellence in Linguistically Diverse Elementary Classrooms

Jessica Guo, Mary Truxaw, and Rebecca Eckert

> "…there is a grain of discovery in the solution of any problem. Your problem may be modest, but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery." (Pólya, 1945/1985, p. v).

When George Pólya wrote these words over 70 years ago, he advocated for rich mathematical problem-solving experiences that challenged curiosity, rather than simply focusing on routine procedures. Relatedly, the Common Core State Standards for Mathematics (CCSSM) expect *all* students to develop rigorous math content and practices, including problem solving (Council of Chief State School Officers [CCSSO] & National Governors Association Center for Best Practices [NGA Center], 2010). Similarly, the joint position statement on mathematics education and social justice from NCSM and TODOS recommends cultivating and sustaining "a positive mathematics identity and affect in students as doers of mathematics" and providing "access to high cognitive demand tasks" (National Council of Supervisors of Mathematics [NCSM] and TODOS: Mathematics for ALL [TODOS], 2016).

Emerging bilingual (EBs) students, as they negotiate more than one language, demonstrate problem solving. Research suggests cognitive advantages of speaking more than one language – for example, cognitive flexibility, higher order thinking skills, and *better problem solving* (Hakuta, 1986; Howard, Christian & Genesee, 2004). Indeed, language can be a resource (e.g., Moschkovich, 2013) and can provide "sources of meaning" (Barwell, 2018). Thus, the growing population of EBs should be well positioned to engage in problem solving. However, math instruction for EBs often focuses on procedures and vocabulary rather than linguistically and cognitively demanding practices (Driscoll, Heck & Malzahn, 2012; Moschkovich, 2007, 2012, 2013). There are consequences of compromising mathematical rigor and practices. For example, national assessments report significantly lower math performance for EBs than for overall students in the same grade levels (National Center for Education Statistics [NCES], 2015). Such performance results suggest disparities in educational opportunities.

Discrepancies between EBs' strengths and math performance suggest a need to rethink how we teach math to them. When perceived from deficit perspectives, EBs may not be held to the same high expectations for math as other students. In order to promote excellence and equity, EBs need "access to rich, rigorous, and relevant mathematics" (NCSM & TODOS, 2016) that recognize their competencies and challenge their thinking (Moschkovich, 2012, 2013). Growth mindset (GMS) and productive struggle are aligned with these recommendations and, further, are associated with motivation, resilience, persistence towards learning goals, and overall academic achievement (Boaler, 2013, 2016; Dweck, 2006). We conjecture that focus on GMS and productive struggle, accompanied by opportunities for rich mathematical problem solving, could support excellence and equity in linguistically diverse classrooms. With these ideas in mind, we ask the following research question:

> *How can a focus on growth mindset and productive struggle support mathematical problem solving and rigorous mathematics in linguistically diverse elementary classrooms?*

To address this question, this paper provides: an overview that defines and connects GMS, mathematics, and issues of equity; a description of a GMS action-research project in a linguistically diverse first-grade classroom; lessons learned; and concluding thoughts.

Mindsets, Mathematics, and Equity

A **mindset** is a core belief about oneself and how one learns. Someone with a **fixed mindset** believes intelligence and abilities are unchangeable – that is, one is either good at something or not. Someone with a **growth mindset** (GMS) believes it is possible to grow intelligence and ability through effort; innate qualities are starting points, not endpoints (Dweck, 2006). Importantly, a supportive learning environment can enhance GMS (Boaler, 2013, 2016). Recent research recognizes neuroplasticity – that is, a capacity for brains to grow and change (Boaler, 2016). There is evidence that when people encounter challenges and make mistakes there is increased brain activity; further, brain activity is enhanced by GMS (Boaler, 2016). GMS and productive struggle are aligned with the CCSSM Mathematical Practices – in particular, MP1 – "Make sense of problems and persevere in solving them" (CCSSO & NGA Center, 2010, p. 6). GMS can powerfully impact math teaching and learning, including motivation, reaction to challenge, and responsibility for one's own learning (Boaler, 2013, 2016).

There are persistent negative narratives that mathematics skills and competencies are innate. In contrast, GMS perspectives assert that everyone is capable of learning mathematics with effort and meaningful learning opportunities (Boaler, 2016). However, some critics worry that focusing on GMS (and similar ideas, such as *grit,* Duckworth, 2016) do not consider structural inequities and discrimination that may hide and/or fail to nurture strengths that students possess (Love, 2018; Wormeli, 2018). EB students, as they negotiate more than one language, demonstrate characteristics of GMS; yet, these characteristics may not be recognized or nurtured. We posit that linguistically diverse classrooms can (and should) provide opportunities to help EB students uncover and develop GMS potential not only for language, but also for mathematics.

Growth Mindset Action Research

To provide a sense of what is possible, we describe a small-scale GMS action research project. The context, lesson design, and data sources are described in this section.

Context. The action research project place took place in a first-grade classroom in Eastbrook School (all names are pseudonyms), a K-5, Title I school (i.e., school receiving supplemental federal funds to assist educational goals of students from low-income homes) in the Eastern United States. The state database identified the school population as follows: 79% Hispanic, 85% eligible for free/reduced meals, and 33% English learners (ELs) (EL is a designation for services based on language proficiency assessments; additional students may be bilingual or emerging bilingual). Eleven of the 21 students in the first-grade class came from homes where Spanish was the primary language; seven students were identified as EL students.

Ms. G. (an author of this article) was a Master's intern in an integrated Bachelor's/Master's teacher preparation program. She co-taught the first-grade class three days per week during the 2016-17 academic year. Her co-teacher, Ms. S., was a full-time teacher at the school. They participated in a professional development and research project aimed at enhancing co-teaching and mathematical discourse in linguistically diverse classrooms. Each co-teaching team included an experienced teacher and an intern who had completed student teaching the prior year.

In university coursework, Ms. G. had been learning about growth mindset and mathematics, as well as strategies for teaching in linguistically diverse classrooms. She decided to develop and implement action research aimed at better understanding and improving her teaching practice and her students' educational experiences (Rossman & Rallis, 2003) – specifically, she focused on GMS to improve student perseverance, academic achievement, and enjoyment of learning challenging mathematics. Key components included: teaching students about GMS; providing a classroom environment that reinforced benefits of GMS; and providing opportunities to experience rich mathematical problem solving within a context where productive struggle was recognized and valued. Ms. G. took the lead in planning and teaching seven GMS-focused lessons and three weeks of math challenge station work. Ms. S., her co-teacher, served in various co-teaching roles during the lessons and activities – for example, she assisted during GMS lessons and worked with other students while Ms. G. facilitated the math challenge station.

Growth Mindset Lesson Design. Ms. G. taught seven developmentally appropriate lessons related to the brain, neuroplasticity, and growth mindset. In the first few lessons, Ms. G. taught about neuroscience and plasticity of the brain. Next, students learned about the difference between a fixed mindset and a growth mindset, and how they could adopt a growth mindset. Then, students learned the importance of hard work, making mistakes, and perseverance. Appendix A describes a lesson that involved simulation of neural pathways where students physically represented the neurons and made connections as they shared learning experiences. Table 1 includes examples of topics, objectives, and resources/ methods from the GMS lessons.

Table 1

Examples from GMS Lessons

Example Topics	**Example Objectives:** **Students will be able to …**	**Example Resources & Methods** For all lessons – Supporting visuals and classroom discourse (e.g., whole class, small group and/or partner talk)
The Brain & Grain Growth	• discuss what they already know about the brain • discuss how they can grow their brains	• KWL Chart ("What I **K**now," "What I **W**ant to Know," and "What I **L**earned.") • Read aloud: *Fantastic Elastic Brain: Stretch It, Shape It* (Deak & Ackerley, 2017). • Building modeling clay brain (see Appendix A)
Neural Pathways	• explain how neurons relate to the brain • give examples of things that are challenging for them	• Video: *Ned the neuron: Challenges grow your brain* (see Appendix B) • Neural pathways simulation (see Appendix A)
Mindsets	• categorize phrases that support a growth mindset or a fixed mindset • use hand signals (open/closed) to identify mindset phrases	• Hand signals to represent growth and fixed mindsets (e.g., open, wiggling fingers: open mindset where the brain has room to grow; closed fist: closed mindset where the brain is stuck and can't move.)
Mistakes and brain growth	• explain how making mistakes can make their brains grow. • identify times that they made mistakes	• Read aloud or video: *The Girl Who Never Made Mistakes* (Pett & Rubinstein, 2011)
Power of *Yet*	• discuss things they cannot do *yet*.	• Read aloud: *Giraffes Can't Dance* (Andreae & Parker-Rees, 2016) • Video: Sesame Street *Power of Yet* (see Appendix B)

Supporting Linguistic Diversity. Aligned with recommendations for teaching EBs (and all students), Ms. G. designed activities to be interactive, to make connections to students' lives and learning, to employ multiple modalities (e.g., reading, writing, speaking, drawing, manipulatives, gestures, movement), and to accept written and spoken responses in students' primary language (e.g., García & Wei, 2014; Celedón-Pattichis & Ramirez, 2012; Teemant, Sherman, & Wilson, 2018).

Data Sources. Ms. G. collected and analyzed qualitative data including: GMS lesson plans, video recordings of GMS lessons and challenge station work, student work samples, observational field notes, interviews of selected students (mix of ethnicity, gender, etc.), and her own reflections. To triangulate qualitative data, she administered surveys to students prior to and after the focused GMS experiences. The survey was designed to gauge students' attitudes about intelligence, mistakes, hard work, and challenges. See Appendix C for survey details.

What We Learned

Building on assumptions that EBs demonstrate traits of GMS in their daily lives, we share and discuss what we learned in one linguistically diverse classroom that supported students in uncovering and developing GMS and productive struggle related to mathematics. Our hope is to inspire others to help their students to recognize, develop, and use their potential more fully.

Challenge Station. To focus specifically on math, Ms. G. set up and facilitated a math challenge station for her students to experience rich mathematical tasks that were designed to create opportunities for curiosity, challenge, collaboration, connection making, and creativity. The challenge station was part of a rotation of math stations (Ms. S., the co-teacher, facilitated other math stations). All students participated in the challenge station every couple of days over the course of three weeks. Ms. G. began each small group session with reminders about GMS (e.g., working hard, making mistakes, and tackling challenges grow our brains). She posed the problem, read it, and discussed it with the students. While students explored the problem, Ms. G. observed, listened to, and interacted with the students and encouraged them to interact with each other (e.g., partner talk). She conferenced with students (e.g., asking questions about their thinking and feelings); adapted problem contexts (as needed) to make them more meaningful and accessible; offered feedback; facilitated modeling with visuals, manipulatives, and acting out problems; celebrated productive struggle (e.g., offering high fives and cheers when mistakes were recognized); and provided time for students to struggle, to think, and to practice problem solving. Table 2 shows examples of challenge station tasks and their sources.

Aligned with Boaler's (2016) recommendations for rich math tasks, challenge station tasks included opportunities for *low floor, high ceiling*; *openness*; *inquiry*; *visuals*; and *convincing and reasoning*. For example, the Block Towers task was *low floor* in that students could easily make at least one 3-block tower; was *high ceiling* in that students could stretch to make multiple 3 block towers and/or towers with a greater number of blocks or different color combinations; was *open* by allowing for multiple methods, pathways, and representations; provided opportunities for *inquiry* to explore combinations and identify patterns; included *visual* (and hands-on) components; and asked students to *convince and reason* as they explained their thinking verbally and in writing/drawing. Other challenge station tasks had similar rich math task characteristics.

Observations. Ms. G. observed that over time many students began to share their mistakes as opportunities to learn and also persevered with challenging math problems. To illustrate, we share some of Ms. G.'s observations about María, a Latinx student from a Spanish-speaking family (not identified by the school as an EL). Early on in mid-February, María was somewhat tentative in demeanor and lacking in engagement. Ms. G. observed: "After introducing the

Table 2
Example Challenge Station Tasks and Sources

Example Tasks	Sources
Block Towers: Students are asked how many different block towers can be made with 3 different colored blocks (and then 4 colored blocks).	Adapted from 3 Blocks Towers Task at https://nrich.maths.org/137 and https://www.youcubed.org/tasks/3-block-towers/
Number Combinations: Students are asked to find different ways to make particular sums (decomposing numbers).	This task builds from the Block Towers task, but uses numbers added together to find particular sums (e.g., sums of 6, 10, and 12)
Digging Dinosaurs: Students are asked to find possible number of 2 or 4 legged dinosaurs based on a picture of 8 legs shown underwater.	Adapted from *Inside Mathematics* Problem of the Month: Digging Dinosaurs, https://bit.ly/3tC2h7t

challenge center … she seemed very quiet and shy… Body language was slumped." However, with experience and encouragement, María demonstrated awareness of a problem's challenges, her own mistakes, and her willingness to "try again," as shown in Ms. G.'s notes from early March:

> When I asked her what was challenging about this station, she said, "It was harder towards the end because I kept on building towers that I already had." She carefully compared each new combination she had to blocks she had already built. When she realized that she had a duplicate, she was not upset, she just kept trying. She would work off of what she already had, instead of starting over. I heard her say, "*Ooo! I made a mistake, this one is the same – let me try again*."

María demonstrated productive struggle and a capacity for problem solving – suggesting that she was on her way to being an active "doer of mathematics" (NCSM & TODOS, 2016).

Interviews. Overall, interviews suggested shifts toward recognition and value of GMS. We highlight excerpts from two interviews with Fernando, a Latinx student with Spanish home language (identified as EL), who had moved from Puerto Rico the previous year. First, we share excerpts from Fernando's pre-GMS activities interview with Ms. G.:

Ms. G.:	What do you do when something is really hard? How does that make you feel?
Fernando:	A little frustrated.
Ms. G.:	Why?
Fernando:	Because you have to try and try and that is really, you know… frustrating.
Ms. G.:	Trying really hard is really frustrating? Can you tell me a little more about that?
Fernando:	When I am in a test, it's really hard when I don't know what to do.
Ms. G.:	Would you rather work on something that is easy for you or hard for you?
Fernando:	Easy.

Next, we share excerpts from Fernando's post-GMS activities interview with Ms. G.:

Ms. G.:	Can you tell me a little more about how you feel when something is hard? How does that make you feel?
Fernando:	That makes me feel great because it makes my brain grow.
Ms. G.:	Why does it make your brain grow?
Fernando:	Because my brain is kind of short but it is going to grow, when I do something hard.
Ms. G.:	Since you said that, would you rather do something that is easy for you? Or hard?
Fernando:	Hard because it makes my brain grow stronger.

Fernando seemed to shift his disposition about productive struggle. We contend that he already experienced productive struggle in aspects of his life, but GMS experiences in this classroom may have helped him to uncover and apply them to school experiences. In any case, we enjoyed hearing that he would rather do something hard because it makes his brain grow!

Survey Trends. Consistent with qualitative data, the surveys showed increases in awareness of and appreciation for GMS ideas. Examples: "I like work that I will learn from, even if I make a lot of mistakes" (pre: 9 students; post: 13 students); and, "When something is hard, it makes me want to do it more" (pre: 6 students; post: 13 students). See Appendix C for more details.

Concluding Thoughts

Revisiting the research question, we assert that practices from this first-grade classroom suggest that *a focus on GMS and productive struggle has potential to support mathematical problem solving and rigorous mathematics in linguistically diverse elementary classrooms.* This small-scale action research project does not provide definitive evidence for how best to support EBs with mathematics, but it does suggest practices that are worth pursuing with aims of developing "a positive mathematics identity and affect in students as doers of mathematics" (NCSM & TODOS, 2016, p. 5). Related to these ideas, we reiterate that strengths associated with bilingualism suggest potential for productive struggle and growth mindset that could support mathematical problem solving. We also repeat the concern that mathematics instruction for EB students too frequently focuses on procedures and vocabulary. The action research suggests that teachers can work to capitalize on the strengths of their linguistically diverse students to press for rich mathematical problem solving.

To impact equity and excellence in mathematics, the lessons and activities described in this article are only beginnings. We recognize that it is important to demonstrate positive, growth mindset attitudes *every* day. It is also critical that students have opportunities to engage in rich mathematical problem solving within the context of classrooms that are linguistically responsive. Based on this action research and our experience, we advocate that teachers work to build from students' strengths outside the classroom in order to promote growth mindset and productive struggle within the classroom. We believe that such focus could help students to experience "the tension and enjoy the triumph of discovery" (Pólya, 1945/1985, p. v). with mathematical problem solving. We believe that such experiences support equity and excellence in mathematics and we invite you to browse the resources in Appendix B to support the journey.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What are your thoughts about how emerging bilingual students are positioned to engage in productive struggle and problem solving? Have your ideas shifted after reading this article?

2. Many of the practices described in this article could be considered "just good teaching." How do they relate to issues of equity and excellence – particularly related to emerging bilingual students? (or do they?)

3. What are next steps for you related to using and/or investigating growth mindset ideas to support your students – especially students in linguistically diverse classrooms?

4. What ideas do you have for helping teachers, students, and school systems to recognize that all students should and can persevere; should and can problem solve, and should and can do rigorous mathematics?

5. As a "try this," we encourage you to explore the resources in Appendix B, discuss them with others, and share other resources that you come across. For example, you might choose a video to view together or rich problems to solve and then to discuss.

Appendix A – Examples from Growth Mindset Lessons

Brainy

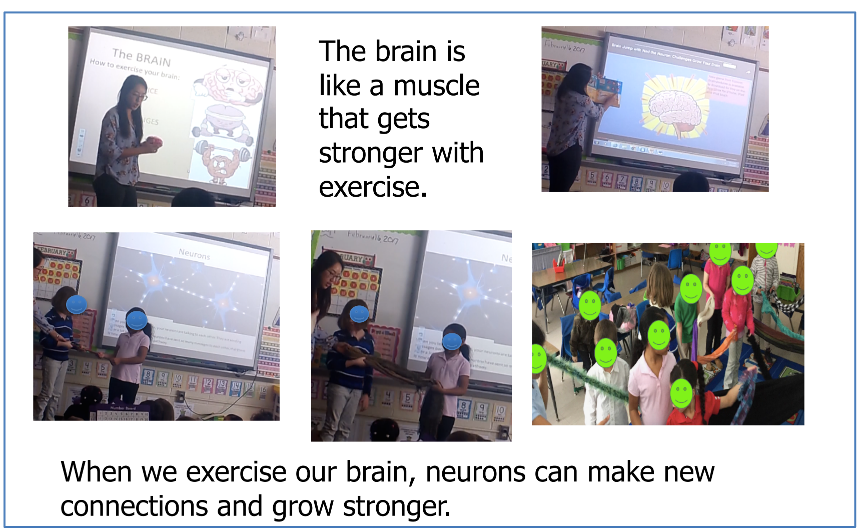
As noted earlier, the first few GMS lessons helped students to learn about neuroscience and plasticity of the brain in developmentally appropriate ways. "Brainy" (made of modeling clay) was an example resource listed in Table 1. In Figure 1, Fernando (mentioned in the article) shows Brainy in the process of being developed. With new learning and challenges, additional coils of modeling clay were added. To show that mistakes "light up the brain," lights were added and that could be blinked on to celebrate mistakes and productive struggle.

Figure 1
"Brainy" (made of modeling clay) was introduced while learning about the brain.

Neural Pathways Lesson

An example lesson mentioned in the article involved a simulation of neural pathways. To begin, Ms. G. asked two students to stand in the front of the classroom to represent neurons, explaining, "When you learn, your neurons are talking together and making connections. When you learned 1+1… your neurons made a connection." Ms. G. gave each student one end of a string to represent connections between the neurons, explaining that thought and learning move through the connections. As they shared other learning experiences, more strings were added to model more neural pathways. As more learning experiences were shared, more students (neurons) joined and strings were replaced with thicker "connections" (e.g., scarves instead of strings). Eventually, all students were connected by the "neural pathways." See Figure 2 for related photos.

Figure 2
Images from growth mindset lessons. Note: Happy faces are used to ensure student anonymity.



Appendix B – Resources

- See the article's reference list for relevant readings.

- Jo Boaler's (2016). *Mathematical Mindsets* book (see reference list) is an excellent mix of research, thought-provoking, and practical ideas related to GMS and math.

- **Youcubed:** https://www.youcubed.org/ This website provides excellent resources to support mathematical mindsets, including videos, tasks, resources, research, and courses. A few examples from the website follow:
  - **Jo Boaler's TedX talk** related to mathematical mindsets: https://www.youcubed.org/resources/jos-tedx-talk/
  - **Growth Mindset Card:** https://www.youcubed.org/wp-content/uploads/2017/03/Mindset-card-with-logo.pdf
  - **Mindset-boosting Videos:** https://www.youcubed.org/resource/mindset-boosting-videos/

- **NRICH enriching mathematics**: https://nrich.maths.org NRICH offers many excellent suggestions and resources for supporting rich mathematics for students.

- **The Mindset Kit**: https://www.mindsetkit.org Free online resource that includes GMS mini-lessons (short videos), sample lesson plans, discussion boards, links to articles, activities, etc.

- **Example Videos for Students:**
  - *Brain Jump with Ned the Neuron: Challenges Grow Your Brain* https://www.youtube.com/watch?v=g7FdMi03CzI
  - *Sesame Street do Growth Mindset* https://www.youtube.com/watch?v=SnrHZ_uvtxk
  - *Sesame Street: Power of Yet* https://www.youtube.com/watch?v=XLeUvZvuvAs
  - *Growth Mindset for Students* – Episode 1/5 (Class Dojo) https://www.youtube.com/watch?v=2zrtHt3bBmQ
  - *Meet the Robinsons—You Failed!* https://www.youtube.com/watch?v=AWtRadR4zYM

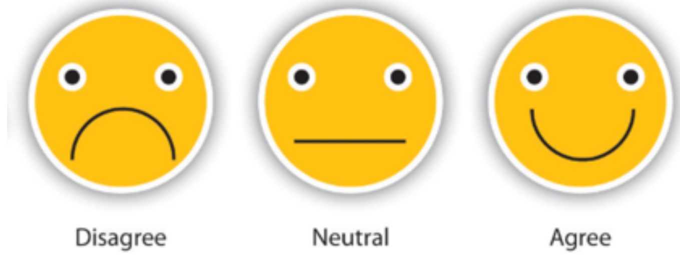
- **See Read-aloud Books in Table 1 and the Reference List.**

Appendix C – Pre- and Post-Survey Information

The survey was adapted from one available at MindsetWorks, a company co-founded by Carol Dweck (https://www.mindsetworks.com/). The original survey was intended for people aged 12 and older so Ms. G. simplified the wording, included picture icons, and asked the questions orally – to make it more developmentally appropriate for first-grade students in this linguistically diverse classroom. The questions used are listed below. The questions followed by asterisks (**) are ones that had more variability than the others.

First-Grade Growth Mindset Survey Questions

Suggestion: Focus on highlighted statements (**) because they seemed to have more variability across student responses.



- I am smart.
- I can change how smart I am.
- I like doing work that makes me think hard.
- I like doing work that is easy for me.
- I like work that I will learn from, even if I make a lot of mistakes. **
- I like my work best when I can do it perfectly without any mistakes.**
- When something is hard it makes me want to do it more.**
- When something is hard I don't want to do it, or I give up. **
- When I work hard it makes me feel like I am not smart. **

The surveys were administered to all students for whom consent forms were received and who were available on the days that the surveys were administered. Due to absences or incomplete pre- or post-surveys, data for 16 of the 21 students were analyzed. Although the sample was small and the intervention was relatively short-term, the data (surveys, interviews, observations, and student work) suggest positive trends for enhanced awareness of GMS and productive struggle. For example, there were shifts in the number of positive responses to survey questions such as, "I like work that I will learn from, even if I make a lot of mistakes," (pre: 9 students [56%]; post: 13 students [81%]) and, "When something is hard, it makes me want to do it more" (pre: 6 students [38%]; post: 13 students [81%]). All survey items showed decreases in fixed mindset responses; all except one survey item showed increases in growth mindset responses, with one response staying the same.

POETRY CORNER

In his latest mathematical poem that begins on the next page,
Lawrence Lesser (The University of Texas at El Paso) makes serious and playful reflections
about cancelling in the school mathematics curriculum,
with additional connections to our greater world.

₵AN
₵EL

Lawrence Mark Lesser

Cancel is from Latin for ‘make like a lattice’,
like crisscrossed wood fencing
in our backyard where we safely
dine with friends,

or like COVID-caused crossouts
on calendars--
a cancelled appointment (*dis-appointment*)
or music event (*dis-concerting*).

Teachers don’t like saying ‘cancel’
lest students get carried away,
cancelling sixes of 26/65,
which *does* equal two-fifths

but it’s ‘cause we multiplied by 1/13 over 1/13,
another name for one.
And don’t say we ‘reduced to lowest terms’
lest students think it shrank.

But context factors into when
  cancelling simplifies:
2/5 is less clear than 26/65
for the chance of drawing a black card
from a deck augmented by another deck’s
  diamonds

while with $y = (x^2 - 1)/(x - 1)$,
cancellation reveals
the whole
limit at $x = 1$.

Now cancelling an x from both sides of $x^2 = 3x$
yields one solution, but loses the other.
Never cancel something
that could be nothing.

My red-voting uncle says our national petri dish
of polarization yields ‘cancel culture’
that cancels minority opinion
not from a minority,

forming a fractious fraction of
conformity over free speech,
to shut up and shout down
undesired values.

Of course, algebraic
structures allow
cancellation
from the left *and* from the right.

Just as we should be slow
to write off students
who casually make a
cancelling error,

who take something like $(x^2 - 8)/(x - 2)$
and strike parts of the expression
without considering
whole factors,

I say don’t always
cancel a person for the terms they use.
Let’s first try calling them *in*,
not calling them out,

and help them see the error,
learn what led them to make it,
and help them outgrow
and correct it

as headphones
make opposing waves
to cancel noise
and lower the volume of the world.

Engaging Bilingual Mathematics Learners With Principle-Based Instruction

Sarah Roberts
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Abstract

This paper uses a framework of four key principles and considers the research question: "How did four middle school mathematics teachers' instruction align with these principles to engage bilingual mathematics students in mathematical work in meaningful ways?" Findings from qualitative methods provide examples from teachers' practice, demonstrating how they: (1) used bilingual learners' funds of knowledge and resources; (2) provided bilingual learners with cognitively demanding work; (3) provided bilingual learners opportunities for rich language and literacy exposure and practice; and (4) identified academic language demands and supports for bilingual learners.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What are meaningful ways to engage bilingual students in both mathematics and language before, during, and after lessons?
2. What goals should mathematics teachers have when working with bilingual students in their mathematics classrooms?
3. What are the key principles mathematics teachers should have for working with bilingual students in their mathematics classroom? How would instruction differ with and without such principles?

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Engaging Bilingual Mathematics Learners with Principle-Based Instruction

Sarah Roberts

As the number of bilingual students across the US increases, there is an urgent need for all mathematics teachers to be prepared to provide meaningful learning environments for these students. Despite this need, bilingual students are often in classrooms where they have few opportunities to learn rich mathematical content (Iddings, 2005; Planas & Gorgorió, 2004). Simultaneously, the mathematics teaching community (e.g., National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) is calling for all students to be immersed in rich mathematical work. However, teachers often struggle with how to approach such work, particularly for bilingual students. This paper uses a framework of four key principles to engage bilingual mathematics students in mathematical work in meaningful ways and considers this research question: "How did four middle school mathematics teachers' instruction align with principles to engage bilingual mathematics students in mathematical work in meaningful ways?"

Theoretical Framework

This study is organized around four key principles drawn from prior work of scholars in the field of mathematics education. These principles support teachers to engage bilingual mathematics students in mathematical work in meaningful ways (Roberts & Bianchini, 2019; Roberts et al., 2017). These principles are understood as reinforcing and overlapping. The first principle is: *Use bilingual learners' funds of knowledge and resources* (Lee et al., 2008; Moll et al., 1992; Moschkovich, 2002). In using this principle, teachers identify, celebrate, and use the knowledge and skills of students, their families, and their communities during mathematics teaching. For example, a teacher might use a student's home language to support mathematics instruction.

The second principle is: *Provide bilingual learners with cognitively demanding work* (Stanford Graduate School of Education, 2013; Tekkumru-Kisa et al., 2015). Here, teachers ensure that bilingual learners can engage in the same kinds of activities and assignments often singularly reserved for those students who only speak English (Iddings, 2005; Planas & Gorgorió, 2004). For example, teachers should ensure that they do not provide an assignment with less cognitive demand for their bilingual students. Teachers should focus on engaging students in the mathematical practices, while balancing conceptual understanding and procedural fluency (Moschkovich, 2013).

The third principle is: *Provide bilingual learners opportunities for rich language and literacy exposure and practice* (Khisty & Chval, 2002; Lee et al., 2013). For example, teachers could create opportunities for bilingual students to receive comprehensible input through listening and reading and provide opportunities for bilingual students to produce comprehensible output through speaking and writing. Opportunities for communicating mathematical ideas should be emphasized over low-level language skills (Moschkovich, 2013).

The fourth principle is: *Identify academic language demands and supports for bilingual learners* (Aguirre & Bunch, 2012; Lyon et al., 2016). For example, teachers could attend to the language demands in the tasks they implement, providing appropriate supports, such as sentence frames, so that all students could share their ideas and reasoning in whole class and small group discussions.

Method

Context

This research took place in the Western United States in a large metropolitan school district with approximately 37,000 students. Within the district, 38% of the students were classified as "English learners[1]." These students came from over 135 countries and spoke over 115

[1] The author uses the term "bilingual" in the paper to identify students who spoke more than one language to acknowledge the resources of these students; however, the district identified these students as "English learners."

languages, with 86% of the "English learners" being Spanish-speakers. Additionally, 69% of the students in the district received free or reduced lunch. The "English learner" demographics of the classrooms of participating teachers mirrored those of the district, with students in the participating teachers' classrooms classified at varying levels of English proficiency after taking the ACCESS (WIDA ACCESS Placement Test—W-APT, 2020; i.e., non-English proficient [NEP], limited English proficient [LEP], and full English proficient [FEP]). The middle schools in the district all used *Connected Mathematics Project 2* (*CMP2*; Lappan et al., 2009a). The content of the tasks included: finding area and perimeter of rectangles and triangles; working with linear functions and writing equations; and working with fractions, decimals, and percent.

Participants

Four White monolingual English-speaking middle school mathematics teachers (three females and one male) from four different middle schools in the district participated in this study. The teachers had 2.5-6.5 years of teaching experience. Ms. Wilson taught sixth grade, while Mr. Xavier, Mrs. Yost, and Ms. Zelner all taught seventh grade. Pseudonyms are used for all participants. The district chose these teachers as "exemplary" teachers of bilingual students, because of their record of good teaching with bilingual students (i.e., as related to test scores and reputation).

Data Collection

I videotaped 10 class periods of the same class for each teacher over the course of a single spring semester. The mean length of each video was 65 minutes, and the video footage collected totaled 42.75 hours. One camera focused on the teacher, who wore a lapel microphone. I also took detailed field notes during each observation. Teachers were not exposed to the principles during the study; these were instead used as an analytic framework.

Data Analysis

The first step of the analysis process was to create field notes during data collection, noting moments of interest related to how teachers attended to bilingual students. Next, I created content logs of videos (Jordan & Henderson, 1995) to enhance field notes. Then, I created transcripts of videos, reading through the data, identifying key pieces of talk related to the four principles for working with bilingual mathematics students, and noting key ideas and relationships, while also developing tentative ideas about how to categorize data (Maxwell, 2013). Table 1 provides a list of example codes I created in this process. I coded the corpus of data for all four theoretical categories and further differentiated the codes in each category. Following coding, I created a data matrix of the coded data according to each category to look for consistencies and inconsistencies across single and multiple participants (Yin, 2011).

Table 1
Example Codes Developed from Theoretical Categories

Theoretical Category	Example Codes
Use Student Resources and Funds of Knowledge	• Home language • Prior knowledge • Prior experiences
Cognitively-Demanding Work	• Sustained use of high cognitive demand tasks • Focus on student mathematical reasoning • Questioning
Opportunities for Rich Language and Literacy Exposure	• Practice with speaking, writing, representations
Academic Language Demands and Supports	• Helping students produce spoken and/or written discourse • Provide scaffolds for language

Findings

The participants varied in how they engaged bilingual students with the principles. The findings in the next sections provide pivotal examples of practice from the participants' classrooms.

Using Students' Resources and Funds of Knowledge

The teachers used the context of problems in *Connected Mathematics* (Lappan et al., 2009b) as a mechanism to *use students' resources and funds of knowledge*. Across the classrooms, participating teachers made many efforts to engage with students' prior experiences as well as with real-life experiences (Moschkovich, 2002; Stanford Graduate School of Education, 2013), as they were linked to problems in the texts. For example, when the students began a series of problems on planning a bike trip, Mr. Xavier had students discuss what it was like to ride bicycles and here is the opening of that conversation:

> **Mr. Xavier**: How many of you guys have ever ridden a bike? How many of you guys own a bike right now? (Most students raise hands.) Cool. Tell me about your bike experience. What do you like about it? Dameon, what do you like about it?
> **Dameon**: You can do a wheelie.
> **Mr. Xavier**: That we can do what? Do a wheelie? OK.
> **Julio**: You can go down a hill super, super fast.
> **Mr. Xavier**: You can go down a hill really fast.
> **Mateo**: I like when the cool breeze splashes in your face.
> **Mr. Xavier**: Getting a cool breeze. What else?
> **Duante**: That you could have a ride, transportation anywhere you want…It's like having a car without the gasoline.
> (Mr. Xavier, 3/9/12, Lines 51-62)

Following this exchange, students then worked with partners for five minutes to write on paper to complete the following prompts from the text: "How far do you think you could ride in a day? How do you think the speed of your ride would change during the course of the day? What conditions would affect the speed and distances you could ride?" (Lappan et al., 2009b, p. 6). Based on students' prior conversations and the students' own experiences, Mr. Xavier continued to elicit students' own thoughts and experiences.

Once the class got to the middle (the "explore" section) of the class period, Mr. Xavier created a shared experience for the students. In this particular problem, students collected data on jumping jacks. Mr. Xavier had students in groups of four, and every student collected data, did jumping jacks, and tracked the time. Here, Mr. Xavier provided each student with the opportunity to engage in the community and drew on students' resources in this mathematical data collection. Mr. Xavier noted that the next step would be the following: "We'll continue to look for patterns in our table…And that goes back to your objective: Record data and then look for relationships in the table" (3/9/12, Lines 683-685). Students would work together to analyze their data.

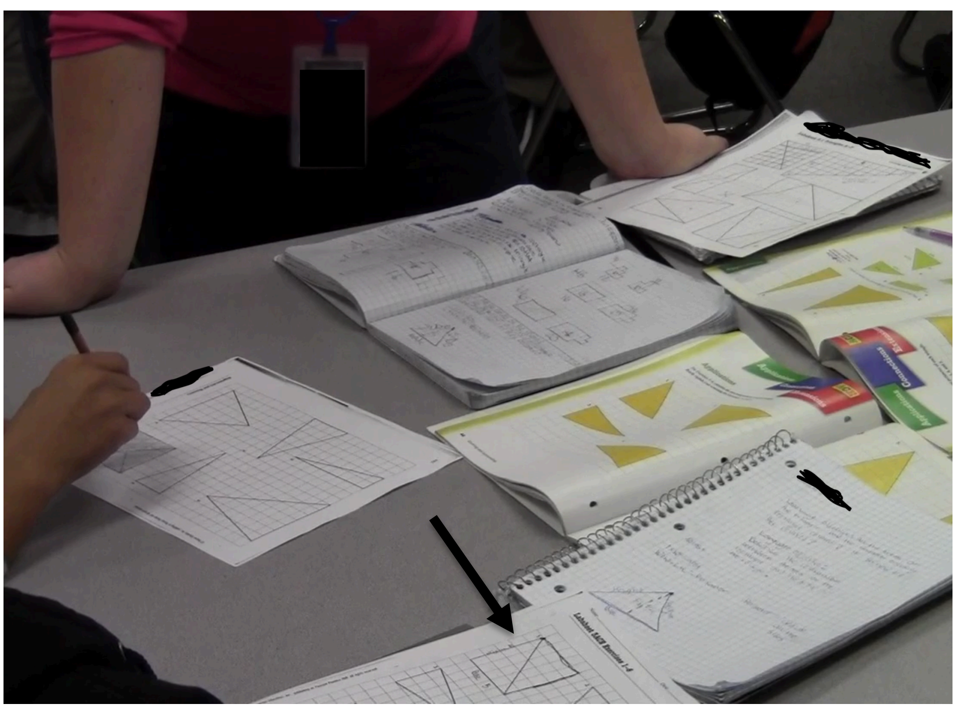
Mr. Xavier initially made connections about how much students knew about bikes, providing connections to students' prior knowledge. Then he created a common experience from which all students could draw while they were completing their mathematics task. He also drew on another language principle, creating an extended rich language opportunity by having students talk about their prior experiences, and he used those experiences to set-up and engage with their textbook problem. In doing this work, students were able to see that their experiences were valued in the mathematics classroom, thereby humanizing the mathematics (Gutiérrez, 2013; Yeh & Otis, 2019); Mr. Xavier provided more buy-in for students to do mathematics in the classroom and for them to make meaning of that mathematics. This work allowed students to begin to understand the meaning of the mathematics around the topic of slope—such as varying speeds and constant speed—which would have been harder for students to understand without shared language and a lived experienced (and discussion of that experience) of the actual phenomenon.

Providing Learners with Opportunities for Cognitively Demanding Work

Mrs. Yost helped students self-regulate or self-monitor as a method for *providing learners with opportunities for cognitively demanding work* in her class (Stein et al., 2000). One method the students used for monitoring their progress was explaining their thinking to their peers and then sharing their thinking with Mrs. Yost. This occurred in a number of settings, such as in summaries of lessons and when peers talked to one another in small groups. An example of this self-regulation occurred during a lesson on developing the formula for area of triangles. Mrs. Yost walked around the room while students worked, and she asked two students, Juan and Teddy, to explain their thinking, as illustrated in the following passage and in Figure 1:

> **Juan**: Don't you divide 25 by 2?
> **Teddy**: There's not (25 squares) in the triangle, [but there is in the rectangle.]
> **Juan**: [It's like you're adding another half to the] triangle.
> **Mrs. Yost**: Listen to what he is saying to you. Say that again.
> **Juan**: That, that you could, you could divide 25 by two, because you're adding one more piece to the rectangle, I mean to the triangle, to make it a half-equal pieces. But then, after you got, after you know how much the pieces here, divided by two….
> **Mrs. Yost**: Okay, so hold on. We have this idea of base and height. So, you're telling me base times height (turns to Teddy), and then you're telling me that I have to cut it in half (turns to Juan) or divide by two.
> **Juan**: You could, the rectangle strategy is going to be, and if, because you know that there is, like, 25 (unit squares), and this is half of the rectangle, so if you do 25 divided by two, that would give you the are-, the space of, the.
> **Teddy**: That would give you the triangle.
> **Juan**: So, yeah, you add the rectangle, and then you do that.
> **Mrs. Yost**: You do what?
> **Juan**: You do 25 divided by two.
> **Mrs. Yost**: OK, so how did I get to the 25? What did I do? So, let's not use any more numbers. Let's take our numbers out for a second, and let's start using some math words.
> **Juan**: You did base time height.
> **Mrs. Yost**: Okay, so I had the base times the height. Then that gave me 25. And then what did I do?...And why did dividing it by two makes sense?...Well, that works for this triangle. Is it going to work for all triangles?
> Note: [ ] denotes overlapping speech, and … denotes omitted talk.
> (Mrs. Yost, 1/25/12, 36:50-38:30)

Figure 1
Mrs. Yost, Teddy, and Juan working through Area of a Triangle



Note. Juan (holding a pencil) is explaining to Teddy the need to divide the rectangle (where the arrow points) by two.

Mrs. Yost supported these students to self-regulate their thinking and check their own thinking for accuracy by having Juan and Teddy explain their thinking related to finding the formula for the area of a triangle. Mrs. Yost focused on the mathematics that students were working through and, in turn, she also created a rich language opportunity. Mrs. Yost's questioning played a key role in her maintenance of the cognitive demand while supporting these students' self-monitoring and self-regulation. NCTM (2014) notes the importance of posing purposeful questions. This questioning helped to immerse students in the mathematics further and provided students with an expectation that they would discuss mathematics while working in small groups. It is notable that Mrs. Yost focused on getting students to talk about the mathematics, even if their mathematical language was still developing. It was not until the end of her turn with the students that she told them, "Let's take our numbers out for a second, and let's start using some math words."

Opportunities for Rich Language and Literacy Exposure

Each teacher in this study tried to provide students in their classroom with *opportunities for rich language and literacy exposure and practice*. For example, as noted above, both Mr. Xavier and Mrs. Yost used large and small group discussions. Opportunities for rich language and literacy were similarly present in Ms. Wilson's classroom during a data analysis lesson. She had students write sentences about data toward the end of the school year:

> [C]an you tell me all about Jasmine's [a character in the problem] reaction time? What are you going to tell [your language arts teacher]? If [your language arts teacher] can't see the tables and graphs, what are you going to tell [your language arts teacher]?
> (Ms. Wilson, 5/1/12, Lines 459-460)

Students first brainstormed, as a class, all the words they knew for comparing data, and wrote all these words on the board at the front of the room. They then worked in groups to create data comparisons that included tables, graphs, and written sentences.

The next day, Ms. Wilson selected student work and shared some of the student sentences with the whole class. The class worked together on the document camera, with Ms. Wilson helping the class work to make the sentences clearer through general editing, clarifying terms, and using more formalized mathematical terms in some cases. For example, in the following interaction, the class revised a pair's sentence about reaction times:

> **Ms. Wilson**: "The reaction Nathanial gave in Trial two was the quickest as opposed to Trial four and five where it took the longest to react."…What details do you want to add?...Where do you want me to put it?...Marquis thinks we need to add more details, like how quick was Trial two. Where should we add that? What should we do?....70…"The reaction time Nathanial gave in Trial two, (Teacher adds a comma) .70 was the quickest"…Can you just stick a number in or do you have to describe it?...Is it .7 hats? Is it .70?
> **Jason**: Milliseconds.
> **Ms. Wilson**: Seconds…Let's just say seconds right now. Don't forget the milliseconds for now. "The reaction time Nathanial gave in Trial two, 70 one-hundredths of a second was the quickest as opposed to Trial four and five where it took the longest to react." Anything else we can add?...
> **Marquis**: You could add the slowest reaction time.
> **Ms. Wilson**: What was the slowest reaction time?
> **Mateo**: 93 hundredths.
> **Ms. Wilson**: .93….The reaction Nathanial gave in Trial two , 70 hundredths of a second was the quickest, as opposed to Trials four and five, where it took the longest to react. Where do you want to add 93 hundredths of a second?...I like this sentence because…it gives me data. Does it tell me the trials?...Does it tell me the times?...And does it tell me about the story? Does it use quickest, slowest, reaction? So, it tells me the data and it tells me about the data. Good sentences.
> (Ms. Wilson, 050212T, Lines 666-719)

Ms. Wilson worked with the students to develop their academic language over the course of two class periods, developing both their mathematics and their language in this final phase while revising their sentences and attending to both their mathematics and language. She provided opportunities for students to use multiple modes of communication and mathematical representations, engaging students in a rich language opportunity and including academic language demands and supports. Ms.

Wilson had students examine data in tables and graphs, discuss the relationships they noticed, write sentences about the data relationships, and then work as a whole class to analyze these relationships and sentences—to have multiple modes of communication and multiple opportunities to examine the mathematics in the problem. Finally, Ms. Wilson provided student work exemplars, supported syntax, and facilitated a whole class discussion (Hakuta et al., 2013; Zwiers et al., 2014).

Academic Language Demands and Supports

The teachers provided a variety of *academic language demands and supports* for their students. For example, Ms. Zelner used practices associated with sheltered content instruction (Echevarria & Graves, 1998), such as having all her students speak in complete sentences (a practice with which all participants engaged their students) and providing students with gestures to understand concepts like x- and y-axes. She also occasionally provided students with sentence frames to support their writing, such as these for interpreting graphs:

> The independent variable is __________________.
> The dependent variable is ____________________.
> The graph that matches this story is ____________.
> A story for this graph is that __________________.
> (Ms. Zelner, 3/8/12, 2:15)

Ms. Wilson also used many sheltered instruction strategies, like pre-reading a text, and she tended to use academic language supports to develop content knowledge. While students learned about probability, Ms. Wilson used a number of general academic language supports within a single lesson to engage students with the content: using a whole class discussion, making sure that all students participated, using multiple mathematical representations, skimming through the text to get a sense for upcoming content, and using models of sentence writing.

The following example draws on many scaffolds from the first day of the probability unit, where Ms. Wilson had students familiarize themselves with probabilities of events and how they would state such likelihoods. Ms. Wilson also used these scaffolds to provide students with language rich opportunities, which engaged students in discussions with one another. She began by having students explore their new text. Then Ms. Wilson modeled what she expected from students, in an example of an academic language support. Ms. Wilson had students share the outcomes of events and prompted them for individual probabilities, finishing with the directions for the task, a probability written as a sentence. (Because of unclear audio, her students' responses were inaudible, as noted at several spots with "…"; however, she did revoice several student responses.) Ms. Wilson began the probability conversations in the following way:

> It has an outcome or a likelihood of something happening. Why between zero and one?
> I mean, can't the number be 7? I say the chances of it happening are 7. Does that make any sense?...Can someone else give us another example of a probability, the likelihood of something happening?...A two out of six chance that I'll get a red marble. Anyone else? Lots of probabilities you can come up with. You can make up a story… Okay here's the deal. I'm only getting a couple ideas. You have a full book of ideas of probabilities. You can come up with anything from sports, to school, to home… I need a sentence from you, and you get to make up the number.
> (Ms. Wilson, 2/23/12, 16:15-16:45;17:25-18:07;18:48-19:15)

Ms. Wilson was then able to engage her class in a whole class discussion, and she elicited students' thinking—an ambitious teaching practice (Lampert et al., 2013). When students were able to think through those probabilities and to communicate the mathematics to their peers, Ms. Wilson was able to make students responsible for this mathematical work—not just her, as the teacher. Ms. Wilson also modeled examples of target language before students attempted it (Khisty & Morales, 1999) and monitored the language that the class used (Khisty, 1996) as they provided examples of probabilities of events. Finally, she revoiced student answers (Moschkovich, 1999), giving value to student responses and giving all students a chance to hear what other students had said—clearly and slowly. All this work helped prepare students for the work they then completed in partners with the mathematical text. More importantly, Ms. Wilson's academic language supports with her students went beyond a focus on vocabulary and word problems and provided rich language opportunities.

Conclusions

Research has consistently shown that bilingual learners need opportunities to engage in supportive language rich environments, where teachers help students make sense of the academic language demands and use students' resources. Additionally, we know that bilingual learners are often provided with fewer opportunities to engage in cognitively demanding work. Teachers in this study were often able to draw on multiple principles at once as they worked with their bilingual students. This overlap might provide increased access to mathematics and language; it potentially provides students with more tools in their toolbox to work with rich mathematics and to engage, connect, and discuss mathematics.

Teachers often shy away from language rich tasks for fear that their students will not understand the mathematics and the language, and as a result, bilingual learners often get less access to rich mathematics. However, these teaching examples and the *CMP2* curriculum (Lappan et al., 2009a) reiterate that teachers can provide bilingual learners with rich mathematics opportunities. With this, more research is also needed around working with mathematics teachers of bilingual learners to support them in using these principles to assist their students in engaging with rich mathematics and developing academic language and discourse that goes beyond basic vocabulary.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. How could you apply the principles discussed in the article to work with your own students (or in another classroom)?

2. How do the principles discussed in this article intersect in practice?

3. What principles might be missing from this list? Would you change any principles? Why?

On the Meaning of Young Children's Mathematics Learning

Cristina Valencia Mazzanti
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Abstract

Through events of kindergarten children learning mathematics, I examine how experiences can shape an idea of what mathematics is and what its purposes are. I question how mathematics is experienced by young children in schools, how those experiences may shape students' perceptions of mathematics, and how teachers can disrupt the unintentional messages that mathematics teaching can convey. I pose that mathematics instruction should foster children's understanding of how their worldviews shape their perceptions of the discipline; allowing them to experience mathematics as a resource to problem-solve and think about the realities of the world around them.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. During your years in school, did you have an experience that made you fall in love with mathematics? If so, describe it.

2. What ideas about mathematics are more prevalent in recent mathematics instruction you have implemented or observed?

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On the Meaning of Young Children's Mathematics Learning

Cristina Valencia Mazzanti

> *Ogni comprensione del singolo elemento è condizionata dalla compressione del tutto. Ogni spiegazione del singolo elemento presuppone la spiegazione del tutto.*[1]
>
> \- *Hans-Georg Gadamer*

Beginnings

I don't remember falling in love with mathematics. I remember learning mathematics. As a child, I measured odd things like school staircases, played with Cuisenaire rods in and out of the classroom, learned about the history of fractions, made drawings, and did mathematics in the "real world". Such experiences learning mathematics ingrained in me that numbers are never a neutral representation of quantity; they are socially constructed and take their meaning from our experiences with them. My high school years centered on learning mathematics in connection to complex philosophical, historical, and aesthetic issues. I learned in deep ways that mathematics was primordially a way people had developed to see the world, with its own set of assumptions and pre-established rules. It was then that I realized that I loved mathematics because I came to understand how mathematics could help me create meaning and transform the world. It was then that I experienced mathematics as a deep expression of the ways of being of humans and the world that we live in.

This feeling of being in love with mathematics is still with me and has guided me as I continued to learn about it, transitioning from student, to teacher, to educator and researcher. In recent years, I have observed many other teachers' mathematics instruction and I taught mathematics bilingually to kindergarten students in the role of researcher and volunteer teacher. Indeed, it is through the diversity of experiences learning and teaching mathematics that I have come to be curious about the messages that children receive about mathematics in formal schooling and the impressions children may have of what doing and knowing mathematics is. I have come to wonder why it is that mathematics is so often thought of as a neutral and objective discipline, as a practice that is significant only for schooling purposes, or as an abstract skill that operates with no connection to context and of little use in understanding our social world. I have reflected on how my own experiences created such a different perspective of mathematics, helping me see it as a contextualized resource to make sense of our social world. Thus, I am curious to understand how individual experiences young children have with mathematics build on each other to create many different meanings and purposes for learning it.

Sharing Stickers Lesson

For me, questions about how young children learn about the meaning of mathematics became tangible through a conversation with a group of 22 kindergarten students that I taught two mornings a week as part of my research. The conversation came at the end of a lesson where we were collaboratively working to think of different sharing scenarios that would require subtraction to solve them. As a whole group we were discussing how many out of a group of ten stickers I should share. After I placed the sticker sheet on the whiteboard for the children to see, we discussed if I should share a few or many of the stickers that I had. Several children gave their perspectives in Spanish and English, arguing different points of view and mainly stating that I should share a few because I would want to keep most of the stickers. Some of the children stated that I should share half of them, claiming that it would be fair to share and keep the same amount. Few children argued that I should share most of them because that is what the person I would share with would want. After several children gave their perspectives, we did a quick round where most of the class suggested a specific number of stickers I should share. We finally decided to see how many I would have left if I gave 8 away. This was an opportunity for the children to practice using some of the mathematics skills and concepts they were learning. Once the children concluded that I would have two

[1] Translation from Italian: "Each understanding of a single element is conditioned by the understanding of the whole. Every explanation of the single element presupposes the explanation of the whole."

stickers left, I asked if they thought I would be happy or sad. Most of the class responded by immediately yelling that I would be sad, except for one girl who raised her hand. When I called on her, she said "Feliz porque compartiste" (happy because you shared).

After that conversation with the kindergarten students, I left school with a feeling of clarity. I started thinking more openly about the importance of the ways we present mathematics to children. I recognized the experience of the student who thought sharing would make me happy as different. It resonated as a moment of learning mathematics that afforded connections to the student's past rich experiences and to demonstrate complex thinking about the social world. I pondered if this experience simply emerged from the question I posed about feelings and questioned what contributes to young children's understanding that numbers offer a quantitative representation to think about the complexities that we face as people. As I thought of the lesson as a whole, I also reflected that I had only come to problematize and critically examine what it means to learn mathematics since my role had changed from that of a student to that of a teacher. Thus, I questioned the messages children are implicitly receiving and perceiving about mathematics learning and its purpose in our social world.

Alligator and Flies Lesson

The lesson about sharing with the kindergarten students seemed like a stark contrast from a recent lesson that I had observed that seemed so representative of many lessons I had taught myself and that I saw in classrooms I visited. During the observation, the teacher had set up her mathematics instruction in small groups of about 6 students. The group that was working with the teacher focused on word problems on a worksheet to help prepare the children for district-mandated tests. The teacher effectively coached the children to find correct results using their own mathematical strategies and developing new ones, being particularly attentive to students who had difficulties. It was an enjoyable lesson, it demonstrated many current pedagogical practices for teaching, the children were focused, and the children explicitly stated feeling successful. Still, what stood out for me was the complete lack of sense of the context proposed in the word problems that talked about an alligator eating flies. Neither the teacher nor the children seemed to spare time thinking about the alligator or the flies, their attention was on developing sophisticated representations of quantities through abstract models, numerals, and operations.

The contrast between the two lessons described helped me notice that our work as teachers of young children is often to help them identify the important information in a particular situation; for instance, the concept a lesson may address or simply the purpose of everyday classroom activities. During the small group activity the teacher helped the students identify and practice the skills they would need to be successful during the test. However, when we focus on certain pieces of information, we are also neglecting other pieces of information that may be relevant in other ways. Particularly, mathematics teaching often entails helping children learn the necessary knowledge and skills to be able to understand, represent, and solve mathematically the ideas and abstract patterns we identify in our world. During mathematics instruction, teachers may find necessary to coach children on how to distinguish the relevant or essential information to effectively follow a mathematical procedure or describe a situation mathematically. Yet, this may be sending the message to children that mathematics works mostly abstractly and independently of context.

Further Reflections

To better understand what I mean, it may be helpful to think of an example. For instance, think of a common word problem: Jane has 10 apples, she gives her friend 8 apples, how many does she have left? In this case, we may think the key information for a young child who is learning to solve this mathematically is the information that is necessary to translate the words into a number sentence that they can then solve to find the answer. In other words, we may think that the child needs to be able to separate the quantities (10 and 8) from the rest of the information and be able to identify how those quantities relate to each other through a mathematics operation (in this instance subtraction). There is probably little purpose for the child to think about Jane or the apples. Although approaches like this may support the child in being able to effectively and accurately perform a mathematics process, they may also inadvertently communicate that mathematics implies finding the pattern, thinking abstractly, and not focusing on the context that could actually make the task relevant. This may communicate that numbers mostly work abstractly and in time this message can easily translate into understanding mathematics as an objective and neutral representation with little connection to context or the experiences children have in their social world.

In the end, the numbers and operations behind the word problem about Jane and the apples are not fundamentally different from those the children and I explored through the conversation about sharing described earlier; both instances require us to subtract 8 from 10 to find an answer. However, I pose that the experience of mathematics for the students in these two scenarios is fundamentally different and each conveys a different message to children about what mathematics is and what its purpose should be. Thus, teachers must consider how the experiences and lessons they facilitate help children understand that the mathematics skills and concepts being learned have a purpose and are useful in the situations they encounter. My experiences in multiple mathematics settings have led me to notice that even children who can carry out mathematics operations successfully and accurately often have difficulty applying their skills in authentic situations or to solve problems. For instance, a child may be able to correctly add 4 and 7, but they may not be able to correctly identify and solve that operation in a word problem or find the total number of materials they need to give to a couple of small groups when it is their turn to be the class helper. Many children are yet to learn how mathematics can be a resource when they encounter everyday dilemmas. There is a need for teachers to create genuine contexts where mathematics skills and concepts can be deployed with agency on the part of the students to make sense of complex situations, contexts where children have freedom to explore how to use different mathematical concepts and procedures with purpose.

The conversation with the kindergarten students about sharing stickers described earlier portrays an experience of mathematics that is not objective or neutral. When the students and I talked about the various ways to distribute the stickers, what was at the center of our discussion was the context surrounding the numbers and the children's past rich experiences. The children were able to experience how the numbers offered a multifaceted representation of the dilemmas that arose from sharing; a representation that allowed them to explore the complexity and implications of the situation. They were able to make connections and see how the number of stickers shared could create a feeling of happiness or sadness, and on a larger scale understand that numbers are not only abstract concepts but resources to read the world that can have important implications. Through the open conversation, the children also had the opportunity to see and experience how their worldviews shaped their perceptions of the numbers and how this may contrast with that of their peers. The lesson materialized my own experience of mathematics as a social practice, creating an opportunity for students to experience it as such.

Since I taught the lesson about sharing with the kindergarten students, I have continued to come back to it as an example that can help teachers identify different factors that facilitate a different experience of mathematics for children. For starters, it may be helpful to consider that the lesson within which the conversation about sharing emanated did not happen in isolation. Over the last few years, I have come to understand that for kindergarten children the issue of sharing is often a relevant one and one that they can easily translate into questions represented through numbers and other mathematical concepts. Hence, sharing was a scenario I presented to the kindergarten students often, creating similar lessons that provided different levels of support from me as the teacher. During the second semester of the school year, the children and I had been exploring multiple mathematics concepts in relation to the idea of sharing. During that year, the children and I thought about sharing in multiple contexts and situations to explore concepts such as addition, subtraction, and comparisons of numbers.

Throughout these lessons, the children explained and explored different perspectives on sharing, drawing on mathematics skills and concepts in their own terms and arriving at their own conclusions. The children were likely to suggest that people should share equally when talking about possible abstract scenarios. However, when it came to be their turn to share, or when we were working with real objects such as stickers or food, they were more likely to take different approaches, such as wanting to keep the most. Children's stances also changed when they shared with someone they were friends with, being much more likely to give away something that they liked or wanted to someone that they cared for. Students were also likely to share a larger amount when both people sharing had something that the other wanted. All of these experiences gave dimensions to the numbers and mathematical concepts we were learning. Each child crafted a personal way to use mathematics skills and concepts, providing a path for them to make connections to their own lived experiences and contrast them to those of their peers.

The focus on mathematics also provided opportunities for the children to explore sharing through new concepts and ideas they were learning as part of the mathematics curriculum. For instance, the number zero proved to be an interesting way for the children to explore their own beliefs and perspectives regarding the limits of their generosity. Throughout the different lessons and conversations, the children quickly discovered that they could use the number zero to describe a situation where they wanted to keep everything that they could. This led the class to have interesting discussions about what was sharing and what was not, and when it was okay to share or not share. More often than not, when a classmate suggested sharing zero another child would explain that was not sharing. Over time the children developed an implicit rule among themselves about not using zero for sharing. There were also more complex conversations about sharing in relation to zero. For instance, when the children discovered that it was possible to give everything away and keep zero, thus challenging their perspective that zero could not be used for sharing.

Sharing proved to be a relevant concept and experience for young students to learn and think about. Yet, it was not just the focus on sharing that created a meaningful mathematics learning experience for students. It was the way sharing was approached as a concept that students were already trying to understand in different contexts, positioning it as a complex experience that was open to multiple perspectives that were appreciated as equally valid. In my work, I learned that there is a plethora of ideas and contexts that could be used to create opportunities for young students to meaningfully learn mathematics such as the conservation of resources, equality and equity, or the purpose of comparisons. Children are actively making sense and noticing the realities around them. When teachers make connections to their students' inherent curiosity, they create an opportunity to learn about the ways mathematics is a social practice itself and a resource to make sense of the world.

Accordingly, another important factor that significantly contributed to creating a different experience of mathematics for students is that the lessons around sharing were happening bilingually, drawing on both Spanish and English as resources for learning and discussing mathematics. The lessons were implemented within a Dual Language Immersion program where children received instruction in both Spanish and English, with formal instruction being predominantly in Spanish. This meant that the children in the kindergarten were bilingual in Spanish and English with varying levels of proficiency. Although mathematics instruction was intended to be in Spanish, I often encouraged the use of both Spanish and English simultaneously during my lessons. I used this as a pedagogical practice to foster children's ability to make connections across languages and to provide an authentic experience of the way languages are used to communicate and understand among bilingual speakers. In this sense, the use of both languages intended to create a comfortable learning space for children, extending and supporting the experience of mathematics as a socially relevant practice that builds and is shaped by the resources and experiences we have as individuals. The open discussions about sharing were not only intended to be an open invitation for children to engage with mathematics in their own terms, they were also spaces where the children could learn from a teacher who actively chose from her languages carefully as she thought about dilemmas.

Creating learning opportunities for children to experience mathematics meaningfully requires teachers to make intentional instructional choices. Through the discussions of earlier examples, we may note how such learning experiences involve purposefully creating a space where children can practice making choices about using mathematics concepts and skills to make sense of the world and resolve dilemmas, designing and modeling dilemmas mathematically instead of just solving them, and having open conversations about and through mathematics. For this to be possible, teachers and educators need to create and identify meaningful contexts from which children can make sense of mathematics, arising from their noticing of children's thinking about it. Equally as important is for teachers to craft learning experiences for children to become experts; creating opportunities for repetition with concepts, skills, issues, and activities. Doing so can support a sense of mastery and comfort for children who initially were hesitant about engaging with invitations into mathematics learning. Repeated engagement can also foster multiple experiences and perspectives of the concepts and skills being learned. More importantly, creating learning experiences for children to understand mathematics as meaningful requires us as teachers to intentionally reflect on what we believe ought to be the purposes of mathematics, what we want to convey to our students through its learning, and invite young students to actively reflect on what they believe ought to be the purposes of mathematics.

I framed this writing with a quote that reminds us that each individual part constitutes the whole and that the whole is always represented in each individual part. To me, the quote evokes the idea that individual experiences we have with mathematics build on each other to teach us what mathematics *is*. The quote also helps us reflect on the idea that each individual experience of mathematics that we construct with children materializes what we have learned and think mathematics is. Thus, the quote is a reminder of the ways people are interconnected; of how single elements come into their full meaning in relation to others, as a part of a larger set or network. As I consider the experiences of mathematics that I shared in this writing, I cannot help but reflect on the importance of thinking of our students as interconnected to their communities. If we think of our students in such a way, then we are also called to think of our students as people in a process of transformative learning; as people who come with past rich experiences to our classrooms and who will have futures beyond those we can imagine as their teachers. In doing so, we can understand anew the importance of helping our students see mathematics as a resource to comprehend the complex realities that we face in our social world and start thinking more intentionally about the meanings we want to help our students find in the practice of mathematics.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What ideas do you think your students have about what mathematics is?
2. What role does context currently play in your mathematics teaching?
3. What messages do you want to intentionally convey to your students about mathematics and its practice?

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