Teacher Identity

A publication of the California Council on Teacher Education

Summer 2020
Volume 47
Number 3

TEACHER EDUCATION QUARTERLY

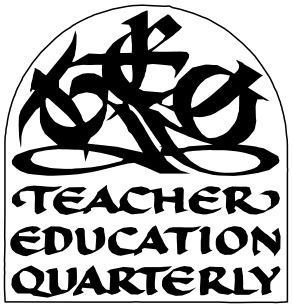
Teacher Education Quarterly

Summer 2020 ◆ Volume 47 ◆ Number 3

Published four times a year by Caddo Gap Press for the California Council on Teacher Education

Published by Caddo Gap Press
Alan H. Jones, Publisher
3145 Geary Boulevard, PMB 275, San Francisco, California 94118 U.S.A.
Telephone 415/666-3012, alanhjones@caddogap.com, www.caddogap.com

ISSN 0737-5328



Dialogic Discourse to Empower Students in Linguistically Diverse Elementary Mathematics Classrooms

Mary P. Truxaw

Abstract

This qualitative research study investigates dialogic discourse (i.e., give-and-take communication where students actively construct meaning) aimed at supporting mathematical meaning making in linguistically diverse elementary classrooms. For this study, linguistically diverse classrooms refer to classrooms where Spanish is the home language of many of the students—both dual-language (Spanish and English) and English-only elementary classrooms that include English learners (ELs). Multiple classrooms in two schools in the western and eastern United States were observed. Data sources include field notes, audio and video recordings, transcriptions of dialogue, and translations of dialogue (as appropriate). Qualitative methods including constant comparative methods, thematic coding, and discourse analysis techniques are used to examine verbal moves, exchanges, sequences, and episodes of discourse from mathematics lessons. A first-grade dual-language classroom is highlighted to illustrate methods and findings. Analysis demonstrates that, similar to monolingual classrooms, specific verbal moves (e.g., exploratory talk, accountable talk, and generative assessment) may shift discourse toward dialogic. Other supporting moves and practices (e.g., think time, visuals, use of primary language, and code-switching) are identified. Models of teaching

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are built from examination of relationships among moves, exchanges, sequences, and episodes within mathematics lessons. The models of teaching demonstrate practices with potential for infusing dialogic discourse and mathematical meaning making in linguistically diverse mathematics classrooms.

Introduction

All students should be provided opportunities to develop understanding of rigorous mathematical content and key mathematical practices, including ones related to problem solving, argumentation, and precision of language (Council of Chief State School Officers [CCSSO] & National Governors Association Center for Best Practices [NGA Center], 2010). However, research has suggested that instruction for the growing population of English learners (ELs; National Clearinghouse for English Language Acquisition [NCELA], 2011) often focuses on procedures and vocabulary rather than linguistically and cognitively demanding activities and meaningful mathematical discourse (Moschkovich, 2007, 2012, 2013), and thus ELs may not be held to "the same high expectations" (CCSSO & NGA Center, 2010) as other students. To develop mathematical proficiency, ELs need empowerment and opportunities to participate in rich mathematical activities and discussions that recognize their competencies, challenge their thinking, and provide necessary support (de Araujo, Smith, & Sakow, 2016; Hakuta, 1986; Moschkovich, 2012, 2013). This study investigates discourse in linguistically diverse elementary classrooms, focusing on potential for dialogic discourse (constructing meaning through dialogue).

Theoretical Framework/Perspectives

Sociocultural Theory and Dialogic Discourse

Sociocultural theory provides a framework for investigating discourse as a mediating tool in the teaching–learning process (Moschkovich, 2002; Vygotsky, 1978, 1934/2012). Consider, for example, applications of Vygotsky's (1978) zone of proximal development (ZPD): "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). The ZPD suggests the need for interactions that support not only current competencies but also *potential* competencies. For all students, including ELs, discourse may mediate *potential* development by supporting back-and-forth processes from thought to word and from word to thought that allow learners to move beyond what might be easy for them to grasp on their own (Vygotsky, 1978, 1934/2012; Wells, 1999).

However, even when the instructional language is the learner's language, the presence of talk does not ensure that understanding follows (Nathan & Knuth, 2003). The *quality and type of discourse* impact its potential for promoting meaningful

mathematical understanding and competencies. In most classrooms, verbal moves are used to transmit information, that is, *univocal* discourse (Lotman, 1988, 2000). In contrast, dialogue that involves give-and-take communication where students actively construct meaning is characterized as *dialogic* discourse. The idea of dialogic discourse stems from Bakhtin (1981) and has been discussed by many others (e.g., Barwell, 2018; Billings & Fitzgerald, 2002; Wells, 1999). For example, Billings and Fitzgerald (2002) described how dialogic discourse has potential to provide "reciprocal flow of ideas involving actions and reactions of group members [that] may lead to new understandings not held by any group member in advance of the discussion" (p. 909). Referring to dialogism, Bakhtin (1981) noted that words become "one's own" only when "the speaker populates it with his own intention, his own accent, when he appropriates the word, adapting it to his own semantic and expressive intention" (p. 293). Dialogic discourse is associated with meaning making (Bakhtin, 1981; Lotman, 2000).

Discourse in Linguistically Diverse Mathematics Classes

When considering linguistically diverse classrooms, dialogue that supports building "one's own" meaning (Bakhtin, 1981) about mathematics is even more complex than such discourse in monolingual classrooms. Too frequently, students in linguistically diverse classrooms are seen from a deficit viewpoint rather than recognizing and building from their competencies (de Araujo et al., 2016) and considering language as a resource (e.g., Moschkovich, 2013) or as "sources of meaning" (Barwell, 2018). Research has suggested cognitive advantages of speaking more than one language, for example, cognitive flexibility, better problem solving, and higher order thinking skills (Anhalt & Rodriguez Pérez, 2013; Hakuta, 1986; Howard, Christian, & Genesee, 2004; Zahner & Moschkovich, 2011). However, it is also true that if one's primary language (L1) is not the language of instruction, and unless the teacher is aware of these nuances, there can be unrecognized challenges involved in switching between languages while learning mathematics (Moschkovich, 2007).

When such challenges are not recognized and not addressed, they can impact students' mathematical points of view and performance (Rojas, 2010). For example, the 2015 National Assessment of Educational Progress results reported that, nationally, only 14% of fourth-grade ELs (as compared to 40% of overall fourth-grade students) were at or above proficient levels for mathematics (National Center for Education Statistics [NCES], 2015). Although focusing on "achievement gaps" can support a narrative that students such as ELs "need to be fixed" (de Araujo et al., 2016, p. 33), it is also true that such "gaps" are indicators of disparities in educational opportunities. That is, such "gaps" suggest that U.S. schools, overall, are not adequately supporting the growing population of ELs.

To provide opportunities for ELs to develop meaningful mathematical proficiencies and practices, it is necessary to keep cognitive demands high (de Araujo et al.,

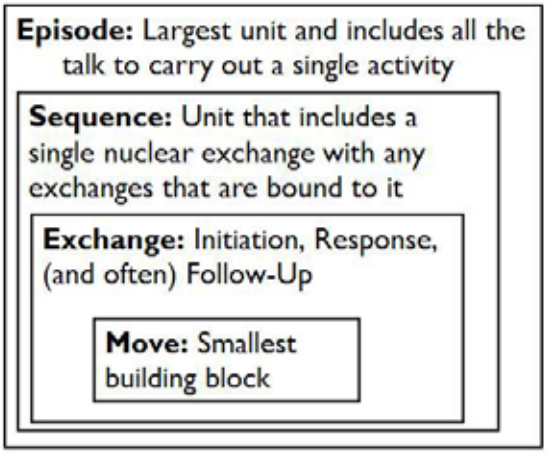
2016; Stein, Smith, Henningsen, & Silver, 2000) but also to make these demands possible through contextual and linguistic support (Cummins, 2005). Along with acquiring mathematical vocabulary, ELs need opportunities to construct meanings for words and ideas within everyday and academic situations and to communicate meaningfully about mathematics (Moschkovich, 2007, 2013), including dialogic discourse that mediates the development of "one's own" meaning. Barwell (2018) discussed relationships between dialogic discourse and multilingual mathematics classrooms when he said, "Mathematical meaning-making happens through the dialogic relations between the diverse discourses, voices and languages that arise, in written and spoken classroom interaction" (p. 162). Dialogic discourse can serve to mediate potential development and press learners to move beyond what they could grasp easily on their own or solely through transmission-style teaching.

Classroom Discourse Analysis

To better understand how these resources and competencies play out to support productive mathematical communication, classroom discourse analysis techniques can be employed. This qualitative research study draws on basic components, structures, and tools adapted from Wells (1999) and Truxaw and DeFranco (2008).

Components of classroom discourse. Wells (1999) parsed classroom dialogue according to the following nested categories: move, exchange, sequence, and episode (see Figure 1). The move is considered the "smallest building block" (p. 236) and is exemplified by a question or answer from one speaker. The exchange, made up of two or more moves, occurs between speakers. Exchanges are categorized as either "nuclear" (if they stand alone) or "bound" (if they are dependent on or embedded

Figure 1
Components of Classroom Discourse (based on Wells, 1999)



within previous exchanges). The sequence is the unit that contains one or more exchanges that are bound together. The episode is made up of sequences, exchanges, and moves and represents all the talk necessary to perform an activity. A lesson may contain one or more episode, depending on the number of activities involved.

Verbal moves. Various categories of verbal moves have been identified within the research literature. For this study, verbal moves are drawn from Truxaw and DeFranco (2008) and are categorized as forms of talk (i.e., monologic, leading, exploratory, or accountable) and forms of assessment/feedback (i.e., inert or generative). See Table 1 for descriptions and examples. Both the forms of talk and the verbal assessment/feedback have been shown to impact the dynamics of discourse

Table 1
Verbal Moves

Verbal Move	*Description*	*Example*
Monologic talk	One person speaking with no verbal response expected (Truxaw & DeFranco, 2008).	Teacher lectures or shares directions without asking for feedback.
Leading talk	Students are led to the teacher's understanding (Truxaw & DeFranco, 2008).	Triadic discourse structure where the teacher initiates a question, student responds, and teacher provides feedback (Cazden, 2001) that leads toward the teacher's point of view.
Exploratory talk	Speaking without answers fully intact, analogous to rough drafts in writing (Cazden, 2001).	Students participate in brainstorming or partner talk.
Accountable talk	Interactions that require accountability to knowledge, to standards of reasoning, and to the learning community (Michaels, O'Connor, Hall, & Resnick, 2002, 2008).	Student offers an explanation that incorporates others' ideas and evidence to support mathematical claims.
Inert assessment	Verbal feedback that tends to maintain the current follow of discourse, supporting tendencies toward univocal discourse (Truxaw & DeFranco, 2008)	"Nice job" or "That is not correct."
Generative assessment	Verbal feedback that mediates discourse to promote students' active monitoring and regulation of thinking (i.e., metacognition) about the mathematics being taught, supporting tendencies toward dialogic functions (Truxaw & DeFranco, 2008)	"What do you think?" or "Why do you think that?" or "Do you agree/disagree and why?"

and the tendency toward univocal or dialogic discourse. For example, monologic talk, leading talk, and inert assessment tend to be associated with each other and also tend toward univocal discourse. Exploratory talk, accountable talk, and generative assessment tend to be associated with each other and also tend more toward dialogic discourse.

Verbal exchanges. The most common pattern of classroom discourse follows the three-part exchange of teacher initiation, student response, and teacher evaluation (IRE) or teacher follow-up (IRF; Cazden, 2001; Mehan, 1985). For example, the teacher could initiate (I) with "What is 7 minus 4?" The student could respond (R) with "It equals 3." And the teacher could evaluate or follow-up (E/F) with "Correct. Good job." This triadic structure has been criticized as encouraging "illusory participation"—that is, participation that is "high on quantity, low on quality"—because "it gives the teacher almost total control of classroom dialogue and social interaction" (Lemke, 1990, p. 168). The teacher's verbal moves (both talk and assessment) seem to influence the function of the discourse. For instance, in an initiating move, the type of question asked has the potential to influence the flow of discourse toward univocal or dialogic. A "quiz-type" question, for example, would more likely lead to univocal discourse, whereas an open-ended question could offer potential for dialogic discourse. Additionally, the follow-up move in the exchange can be pivotal to whether the discourse will tend more toward univocal or dialogic. For example, when the teacher uses the follow-up move as an evaluation, the intended function of the discourse is typically to transmit information (i.e., univocal). On the other hand, if the follow-up move is generative, for example, related more toward an exploratory stance, then discourse has greater potential to be dialogic (Nassaji & Wells, 2000).

Sequences. Sequences involve moves and exchanges that are introduced, negotiated, and brought to completion, making them a unit with "functional significance" (Wells, 1999, p. 236) when analyzing classroom discourse. Truxaw and DeFranco (2008) demonstrated that graphic maps (called sequence maps) could be developed to represent the flow of the verbal moves and the overall function of the discourse (i.e., tendencies toward univocal or dialogic).

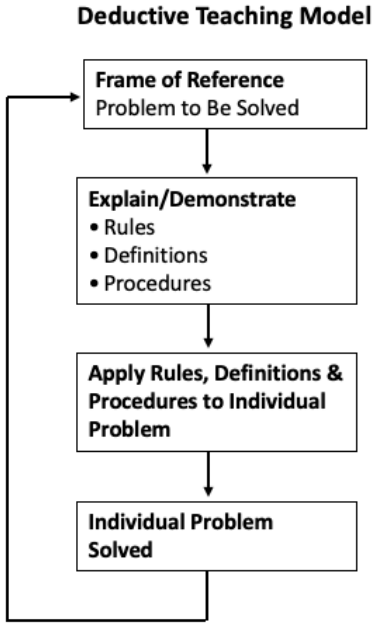
Episodes and models of teaching. Episodes are composed of one or more sequences used to carry out an activity. A math lesson could have one or more episodes. Multilevel analysis (considering moves, exchanges, sequences, and episodes) can be used to develop models that tend to support discourse on a continuum from univocal to dialogic. Two examples adapted from Truxaw and DeFranco (2007, 2008) are described next.

Deductive model. The deductive model (see Figure 2) demonstrates a transmission style of teaching and is more likely to be univocal in nature. For example, the teacher could present a problem or set of problems to be solved (as a frame of reference); explain rules, definitions, and/or procedures (typically using mono-

logic or leading talk and inert assessment/feedback); and then have students apply procedures/rules to the specific problem or set of problems (again, typically using monologic or leading talk and inert assessment/feedback). The focus is on solving individual problems rather than on developing more general mathematical meaning.

Inductive model. The inductive model (see Figure 3) illustrates how discourse can support recursive, inductive cycles rather than linear or deductive steps. The inductive model strategically uses both univocal and dialogic discourse but tends toward dialogic overall (Truxaw & DeFranco, 2007, 2008). For example, the teacher might present a rich problem (as a frame of reference); develop shared meaning through dialogue; and facilitate dialogic discourse to investigate, explore, and examine ideas that move toward new and/or deeper understandings and revised frames of reference. A characteristic of the inductive model is its cyclic nature, where concepts can be visited more than once as the discourse supports revised understandings that are explored and examined. A goal of the inductive model is to move beyond solutions to individual problems toward more general mathematical understanding. The teacher is likely to use a mix of forms of talk and assessment/feedback but strategically infuse generative assessment to press student thinking and provide opportunities for exploratory and accountable talk.

Figure 2
Deductive Model of Teaching (based on Truxaw & DeFranco, 2008)

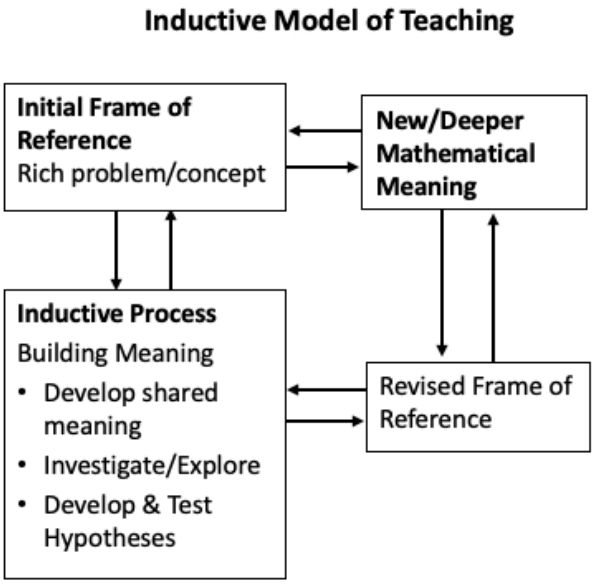


Research has demonstrated that, although there are appropriate times for both univocal and dialogic discourse, dialogic discourse has greater potential for supporting mathematical meaning making (Truxaw & DeFranco, 2008; Wells, 1999). Within monolingual classrooms, interactions of verbal moves within the context of models of teaching have been shown to support discourse on a continuum from univocal to dialogic. Given the importance of supporting and challenging ELs, a logical next step is to similarly analyze discourse in linguistically diverse classrooms.

Research Questions

It is important for ELs to participate in mathematical discourse that challenges and supports both current and *potential* competencies—with opportunities to dialogically build meaning about mathematics (Barwell, 2018; Moschkovich, 2007, 2013; Vygotsky, 1978, 1934/2012). Investigating discourse-rich mathematics instruction in linguistically diverse mathematics classrooms could provide insights to inform related research and practice. With these considerations in mind, this study investigates verbal discourse in linguistically diverse elementary classrooms, focusing on potential for dialogic discourse. Research questions follow:

Figure 3
Inductive Model of Teaching (based on Truxaw & DeFranco, 2007, 2008)



RQ1: How can various forms of verbal moves and supporting moves mediate discourse on a continuum from univocal to dialogic in linguistically diverse mathematics classrooms?

RQ2: How can models of teaching inform mathematics teaching and learning within linguistically diverse classrooms?

Methods

Context

The qualitative research reported in this article is part of an ongoing study investigating mathematics teaching and learning in linguistically diverse classrooms. For this study, the term *linguistically diverse classrooms* refers to classrooms where Spanish is the home language of many of the students. Spanish was selected as the focus language because it is the language, other than English, spoken most frequently in the United States (U.S. Census Bureau, 2013); is the home language, other than English, reported most frequently for students in U.S. schools (NCELA, 2011); and is the most commonly reported home language for ELs (NCES, 2016). Classrooms in two schools (one in the western United States and one in the eastern United States) were purposefully selected because they offered dual-language programs (DLPs) as well as English-only instruction; they also had many ELs among their students.

Garden School. Garden School[1] is a K–5 school located near an urban center in the western United States. The school profile reported the following demographic information for Garden School's students: 92% Hispanic or Latino, 48% ELs, and 85% eligible for free/reduced-price meals (State Department of Education, 2012). The school has a DLP that uses two languages for instruction and learning, Spanish and English. There are two DLP classrooms per grade level, and the remaining classrooms are instructed in English only. Students in the Garden School's DLP classrooms come from primarily Spanish-language-dominant backgrounds. The school principal shared state assessment data demonstrating that the students in the DLP performed higher on state-mandated mathematics assessments than the students in the same school in structured English-immersion classes. There was a strong sense that the school's administration supported the DLP (principal, personal communication, July–October 2012).

East Brook School. East Brook School is a K–5 elementary school located in the eastern United States. The school profile reported the following demographic information for East Brook School's students: 80% Hispanic, 33% ELs, and 85% eligible for free/reduced-price meals (State Department of Education, 2018). The school has a DLP and uses Spanish and English as instructional languages. The DLP at East Brook School is voluntary; families choose to have their children participate

in the program. There are two DLP classrooms per grade level, and the remaining classrooms are instructed in English only. As noted in the strategic school profile, the classes in the DLP are "integrated and balanced" so that each class contains half predominantly English speakers and half predominantly Spanish speakers. According to a state report, the main goals of the DLP "are to enable students to become bilingual, bicultural, bi-literate and, in the process, to reach their highest academic performance."

Data Sources

Observations have taken place in multiple dual-language classrooms in these schools (kindergarten and first through fifth grades) and also multiple English-language-only classrooms (kindergarten and first through fourth grades). Data sources include audio and video recordings, field notes, transcriptions, and translations (as appropriate).

Focus Classroom

For this article, one focus classroom is highlighted to illustrate methods and findings, Señora Castro's first-grade classroom in the DLP at Garden School. Focus data from this classroom were collected in fall 2012. At the time of the observations, Señora Castro had 14 years' teaching experience. She was fluent in Spanish and English and had specialized certification to teach in the DLP, along with elementary teaching certification. The observed lessons were taught in Spanish, but students used both Spanish and English at different times during the day.

Data Coding and Analysis

This study employed qualitative research methods, including constant comparative methods, thematic coding, and discourse analysis. "Constant comparative methods"—that is, the "process of taking information from data collection and comparing it to emerging categories" (Creswell, 1998, p. 57)—were employed to analyze classroom observation data, to triangulate discourse analysis, and to identify practices for supporting dialogic discourse. For example, a *start list* of provisional themes/codes (Miles & Huberman, 1994) was generated from relevant research literature (e.g., practices identified as supporting math discourse and/or ELs). Also, open coding (Creswell, 1998) was used to inductively uncover other categories within the data. Next, axial coding (Creswell, 1998) was employed to look for patterns, interconnections, and relationships across the categories.

Discourse analysis techniques further informed the research, as will be explained next. Multilevel discourse analysis examining verbal moves, exchanges, sequences, and episodes of discourse from mathematics lessons (Truxaw & DeFranco, 2008; Wells, 1999) was employed. Line-by-line coding of transcriptions was analyzed

to identify specific forms of talk and verbal assessment. Figure 4 shows examples of coded text. Column 1 indicates the speaker (e.g., Sra C = Señora Castro, Ss = students); Column 2 shows the original words used (in Spanish); Column 3 shows the English translation; Column 4 shows the move within a triadic structure (I = initiation; R = response; F = follow-up); Column 5 shows the function of the move (e.g., req inf = requests information; inf = informs; eval = evaluates); and Column 6 shows the talk/feedback moves (e.g., Init = initiation; Lead = leading talk; IA = inert assessment).

Coded transcripts were analyzed and used to develop graphic models called *sequence maps* that illustrate the flow of talk and verbal assessment/feedback within sequences and tendencies toward univocal or dialogic discourse (for details, see Truxaw & DeFranco, 2008). Figure 5 shows a brief description, sequence map, and associated dialogue (the same dialogue shown in Figure 4). Each number on the sequence map represents a verbal move—that is, a form of talk or verbal assessment. The flow of the discourse can be tracked by following the numbered moves consecutively. A marker is placed along a line representing a continuum of discourse ranging from univocal to dialogic. To determine univocal and dialogic tendencies, the coded transcripts are examined for indicators compiled from research literature that inform the *overall purpose* of the discourse within a sequence. The sequence shown in Figure 5 is mapped as tending toward univocal since its *overall purpose* is to convey information rather than to generate new meaning.

The sequence maps allow for visualization of patterns of talk and verbal assessment within individual sequences, across sequences within the same lesson, and across cases. Along with graphically illustrating the flow of talk and verbal

Figure 4
Example Coded Transcript

1	2	3	4	5	6
Who	Spanish	English Translation	Mv	Func	Tlk Fdbk
Sra C	¿Listo? Vamos hacer la canción de la mañana. Ready? Go.	Ready? We're going to do the morning song. Ready? Go.	I	req inf	Init, Lead
Sra C & Ss	Buenos días. Buenos días. ¿Cómo estás? ¿Cómo estás? Muy bien gracias. Muy bien gracias. Y usted?	Good morning. Good morning, how are you? How are you? Very well thank you. Very well thank you. And you?	R	inf	Lead
Sra C	Perfecto.	Perfect.	F	eval	IA

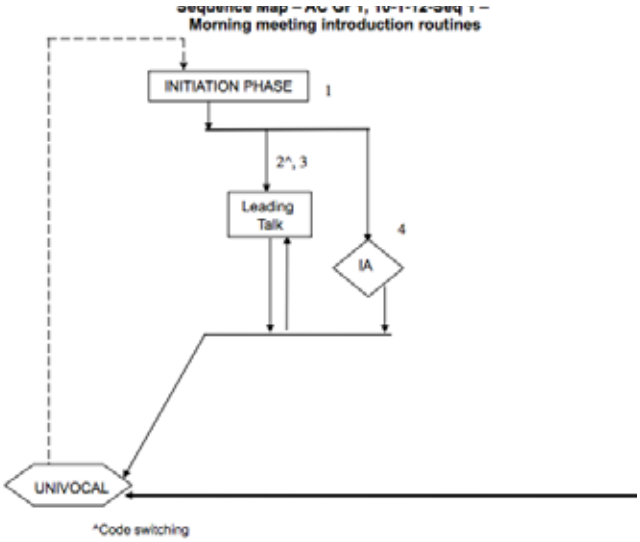
assessment, analysis of sequence maps can suggest situations where episodes would benefit from further analysis.

To answer RQ2, multilevel analysis of identified episodes (and the sequence,

Figure 5

Description, Sequence Map, and Coded Transcript Representing Univocal Discourse

This simple sequence involved a daily routine of singing a song together.
The numbers on the map indicate the verbal moves that coincide with the transcript (far left column of transcript).



Excerpt of Coded Transcript for Sequence 1

# in Map	*Who*	*Spanish*	*English Translation*	*Mv*	*Func*	*Tlk Fdbk*
1 2	Sra C	¿Listo? Vamos hacer la canción de la mañana. Ready? Go.	Ready? We're going to do the morning song. Ready? Go.	I	req inf	Init, Lead
3	Sra C & Ss	Buenos días. Buenos días. ¿Cómo estás? ¿Cómo estás? Muy bien gracias. Muy bien gracias. Y usted?	Good morning. Good morning, how are you? How are you? Very well thank you. Very well thank you. And you?	R	Inf	Lead
4	Sra C	Perfecto.	Perfect.	F	Eval	IA

exchanges, and moves within them) was employed to develop associated models of teaching. Existing models (Truxaw & DeFranco, 2008) provide a base from which to compare and build potential new models of teaching within linguistically diverse mathematics classrooms.

Results and Discussion

Results for RQ1

RQ1 asked, "How can various forms of verbal moves and supporting moves mediate discourse on a continuum from univocal to dialogic in linguistically diverse mathematics classrooms?"

The analysis of the identified verbal moves within linguistically diverse classrooms revealed results compatible to ones uncovered in research in monolingual classrooms (e.g., Truxaw & DeFranco, 2008). Use of monologic talk, leading talk, and inert assessment was more frequently associated with univocal discourse and transmission-style teaching. Use of exploratory talk, accountable talk, and generative assessment was more frequently associated with shifts toward dialogic discourse and more inquiry-oriented teaching. Overall, analysis across the observed mathematics lessons uncovered discourse that was predominantly univocal. However, analysis revealed some instances where there were *shifts toward dialogic discourse*. In these instances, teachers infused generative assessment/feedback that helped to shift from a transmission stance toward a stance where students explored, explained, and/or justified their ideas (using exploratory talk and/or accountable talk). The instances where the discourse shifted toward dialogic provided opportunities for students to exchange ideas, think, and build mathematical meaning. Examples from the focus first-grade classroom will be used to illustrate these shifts.

Analysis using constant comparative methods validated relationships between specific verbal moves and tendencies toward univocal or dialogic discourse. Along with the specific verbal moves, the analysis revealed that shifts toward dialogic discourse were supported by other moves and practices that are consistent with the literature for supporting ELs (e.g., Celedón-Pattichis & Ramirez, 2012; Chapin, O'Connor, & Anderson, 2009; Echevarría, Vogt, & Short, 2007; Moschkovich, 2013). See Table 2 for selected examples.

Illustration of RQ1 results from first-grade math class. One of Señora Castro's first-grade mathematics lessons is used to illustrate results of RQ1. Analysis of the lesson identified four episodes that included a total of 18 sequences (see Table 3). The episodes included a "morning meeting" with a math orientation, instruction and practices with addition and subtraction word problems, independent practice, and closure with whole-group discussion and journal writing. The lesson was taught in Spanish and took place in October 2012.

In the first episode of this lesson (morning meeting), routines with specific

procedures and expectations were facilitated. Although the discourse tended toward univocal, there were times when Señora Castro encouraged students to explore, think, and explain, shifting somewhat toward dialogic discourse. Examples follow. Among their daily routines, the "number of days in school" (in this case, 33 days) was represented in multiple ways—calendar, words, pictures, tally marks, place value materials, and so on. Individual students were asked to complete different representations and then Señora Castro facilitated their sharing. One representation employed sentence frames: "____ es mayor que ____ y es menor que ____" (____ is greater than ____ and is less than ____). The student who completed the sentence frames filled in "33 es mayor que 12 y es menor que 4000000000000" (see Figure 6).

When Señora Castro facilitated sharing of this representation with the class, she responded:

Spanish	**English translation**
Wow. Es mayor que doce, y luego es menor que . . . Ooo! Vamos a ver. Mil, millón, [Adds commas to the number.]	Wow. It's greater than 12, and then it is less than . . . Ooo! Let's see. Thousand, million, este es billón. [Adds commas to the number.]

Table 2
Selected Examples of Supporting Moves

Supporting move	*Description*
Think time	Providing students with time to think about the mathematics (e.g., wait time, partner talk, self-talk)
Visuals	Using visuals (e.g., pictures, writing, gestures, and manipulatives) to support verbal moves
L1 use	Using students' primary language (L1) for instruction
Code-switching	Switching between languages (e.g., English and Spanish)

Table 3
Episodes in One First-Grade Mathematics Lesson – Señora Castro, Teacher

Episode no.	*Topic/theme*	*Sequences*
1	Morning meeting—Day 33 of school—various representations of 33 (calendar, pocket chart, place value sticks, numbers, words, tallies, etc.).	1-9
2	Addition or subtraction ("suma o resta") word problems—whole-class discussion and practice.	10–14
3	Setting up and facilitating independent work.	15
4	Closure—whole-group discussion and journal writing.	16–18

So entonces se dice . . .	So then you say . . .
y este seria trillón. Cuatro trillón.	and this is billions.
Creo que es *four trillion*	and this would be trillion.
[code-switching]. Wow!	I think it is *four trillion*. Wow!
Que numero!	What a number!"

Her enthusiasm for the student's response was supportive, but she didn't stop there. She probed using generative assessment, asking the class, "¿Pero es correcto?" (But is it correct?) to get students to think about the relationships of the numbers. After some discussion, the students and she agreed that the information in the sentence frame was correct.

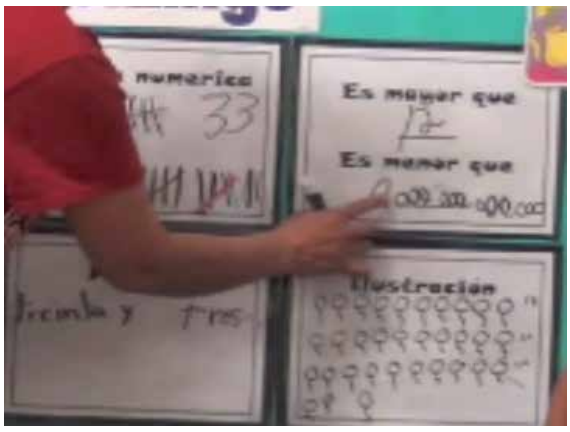
To take a closer look at relationships of transcripts, coding, and mapping, the next sequence in the same lesson will be examined. Sequence 8 took place during the same morning routines while the class was demonstrating and talking about representations for 33 (coinciding with the number of days in school). After sharing various representations for 33, Sequence 8 continued with sharing a representation of 33 using base-ten materials (3 ten rods and 3 unit cubes). Señora Castro employed generative assessment to press student thinking about place value.

Spanish	**English Translation**
Espera. Tengo tres decenas	Wait. I have three tens
y tres unidades ¿Esto es igual?	and three ones. Is this equal?
Tres y tres es igual?	Three and three are equal?
Tres decenas es igual	Are three tens equal
a tres unidades?"	to three ones?

Señora Castro held up the place value blocks for students to see (visuals), holding her hands like a balance (see Figure 7). She asked students to put thumbs up

Figure 6

Señora Castro Pointing to the Sentence Frame Completed by a Student



or down to indicate if they agreed or disagreed that three tens were equal to three ones (using visuals and think time). She then asked several students to explain their thinking, encouraging exploratory and/or accountable talk. Students "explained" with a combination of words (in Spanish, L1 for most of the students) and pointing to materials and numbers (visuals). The students seemed interested and willing to try to use language to build meaning about the place value concepts—exploring the ideas and also demonstrating "accountable talk" through accountability to knowledge, standards of reasoning, and the learning community (Michaels et al., 2008). Señora Castro helped the students to explore and think about the differences between the value of a digit (e.g., 3), the place or position of a digit (e.g., ones place or tens place), and the place value (e.g., 3 or 30), shifting the discourse somewhat toward dialogic (with potential to develop deeper understanding of the place value concepts).

To further illustrate the analysis, Table 4 shows verbal moves, translations, and coding of dialogue (Sequence 8 of the lesson). Figure 8 shows a sequence map that was developed based on the coding seen in Table 4. The numbers on the map indicate the verbal moves that coincide with the transcript (far left column). As described in the "Methods" section, the flow of the discourse can be tracked by following the numbered moves consecutively. A marker is placed along a line representing a continuum of discourse ranging from univocal to dialogic. The coding and map reveal discourse that is predominantly univocal but include some shifts toward dialogic. Within this sequence, the dialogic shifts are associated with exploratory talk, accountable talk, and generative assessment; the verbal moves are further supported by think time, visuals, and use of students' L1 (i.e., Spanish for most of the students). These supporting moves are consistent with strategies

Figure 7

Señora Castro Shows Place Value Materials Representing 33 to Students

Table 4
Verbal Moves, Translations, and Coding of Sequence 8

No. on map	*Who*	*Spanish*	*English translation*	*Move*	*Talk/ feedback*
1,2	Sra C	O, ya se me olvidado algo muy importante. Necesitamos agregar ¿qué?	Oh, now I forgot something very important. We need to add what?	I	Init, IA
3	Ss	Un palito más.	One more stick.	R	Lead
4	Sra C	Un palito más. ¿Se agregaron uno? .	One more stick Did you add one?	F/I	IA
5	S	Tres!	3!	R	AT
6	Sra C	¿Tres más? ¿Qué paso aquí? ¿No agregaron con la substituta?	3 more? What happened here? You didn't add with the substitute [teacher]?	F/I	AT
7	S	No porque mañana era sábado y domingo y viernes.	No because it was Saturday and Sunday and Friday.	R	AT
8	Sra C	O, pero jueves no agregaron?	Oh, but Thursday you didn't add?	F/I	AT
9	S	No.	No.	R	AT
10 11	Sra C	No, okay bueno. Entonces tenemos tres unidades. ¿Cuántos unidades?	No, okay fine. So we have three units. How many units?	F I	IA IA, v
12	Ss	Tres.	Three	R	Lead, v
13	Sra C	Tres. Y se nos (*inaudible) nuestra vasito . . . ¿Cuántas decenas?	Three. And our (*inaudible) little cup . . . How many tens? F	F I	IA, v
14	Ss	Tres.	Three.	R	Lead, v
15	Sra C	Espera. Tengo tres decenas, y tres unidades. ¿Esto es igual? Tres y tres es igual? Tres decenas es igual a tres unidades?	Wait. I have 3 tens, and three ones. Is this equal? Three and three is equal? Three tens is equal to three ones?	F/I	GA, v, tt
16	Ss	[Mixed responses] No . . .	[Mixed responses] No . . .	R	ET, tt
17	Sra C	¿Quien piensa que sí? [gestures thumbs up]	Who thinks it is? [gestures thumbs up]	I	GA
18	Ss	[some ss indicate thumbs up]	[some ss indicate thumbs up]	R	AT, tt
19	Sra C	¿Quién piensa que no? [T gestures thumb down]	Who thinks it isn't? [T gestures thumb down]	I	GA, tt
20	Ss	[some ss indicate thumbs down]	[some ss indicate thumbs down]	R	AT, tt

—continued on next age—

Table 4
Verbal Moves, Translations, and Coding of Sequence 8 (continued)

No. on map	*Who*	*Spanish*	*English translation*	*Move*	*Talk/ feedback*
21	Sra C	Hmm okay, ¿quién nos puede decir porque no? ¿Por qué no es igual? [S name], porque no es igual?	Hmmm okay, who wants to tell us why it isn't? Why isn't it equal? [S name], why isn't it equal?	I	GA, tt
22	S	. . . la suma no puede hacer igual porque hay 30 allí, y si pones tres mas no es tres . . .	. . . the amount can't be equal because there are 30 there, and if you put 3 more it isn't 3 . . .	R	AT, v
23	Sra C	So, este son mas. Sí, so esta son tres unidades.	So, these are more. Yes, so this is 3 ones.	F	AT, v
24	S	También hay de esos, de esos. [S stands up & points to sticks]	Also there are those, from those. [S stands up & points to sticks]	I	AT, v
25	Sra C	Sí, o, so tú, [S name], quieres decir que cada uno vale . . .	Yes, oh, so you, [S name], want to say that each one is worth . . .	R I	IA+
26	S	10.	10.	R	Lead
27	Sra C	10. En realidad esto es 10, 20, 30. . . . 30, y luego . . . cuento, listo? [S name] vamos así.	10. In reality this is 10, 20, 30 . . . 30, and later . . . count, ready? [S name], like this.	F/I	IA
28	Ss/Sra C	10, 20, 30, 31, 32, 33.	10, 20, 30, 31, 32, 33.	R	Lead
29	Sra C	Perfecto, 33 . . .	Perfect, 33 . . .	F	IA

Note. Move column: I = initiate; R = response; F = follow-up.
Talk/feedback column: IA = inert assessment; GA = generative assessment; Lead = leading talk; ET = exploratory talk; AT = accountable talk (accountable to one or more of the following: knowledge, standards of reasoning, or the learning community); v = visuals; tt = think time.

suggested in the literature (e.g., Celedón-Pattichis & Ramirez, 2012; Chapin et al., 2009; Echevarría et al., 2007; Moschkovich, 2013).

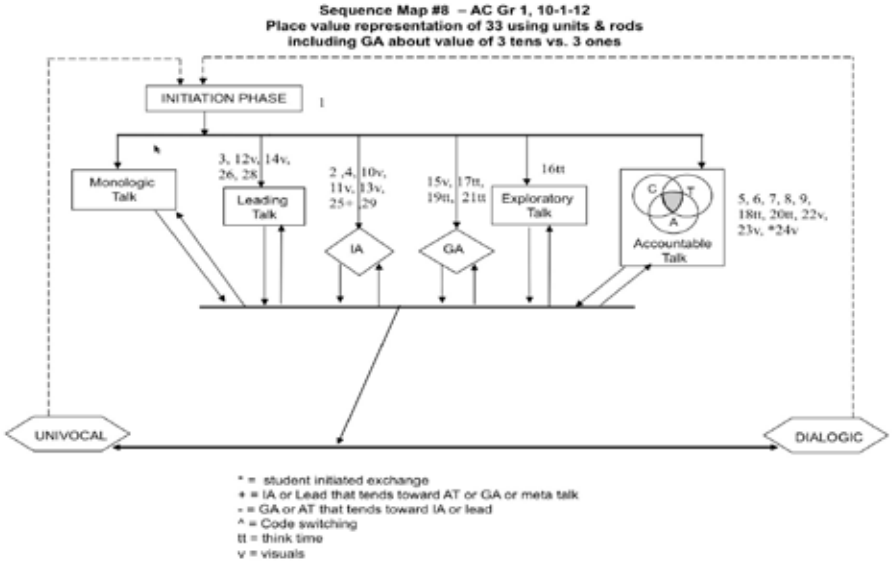
Summary of results related to RQ1. The analysis demonstrates that verbal moves (e.g., monologic talk, leading talk, exploratory talk, accountable talk, inert assessment, and generative assessment) impact discourse on a continuum from univocal to dialogic in linguistically diverse classrooms. Consistent with research from monolingual classrooms (Truxaw & DeFranco, 2008), the verbal moves most frequently associated with dialogic discourse include exploratory talk, accountable talk, and generative assessment. Along with the verbal moves, the current research uncovered additional *supporting moves* that are consistent with the literature that impact the discourse in linguistically diverse classrooms, for example, think time, visuals, use of L1, and code-switching. Although fully dialogic discourse has not yet been uncovered in the observed linguistically diverse classrooms, instances where the discourse shifts toward dialogic provide hints for how to promote more meaningful discourse in these classrooms.

Results for RQ2

RQ2 asked, "How can models of teaching inform mathematics teaching and learning within linguistically diverse classrooms?"

Figure 8

Sequence Map for Sequence 8 of Grade 1 Class

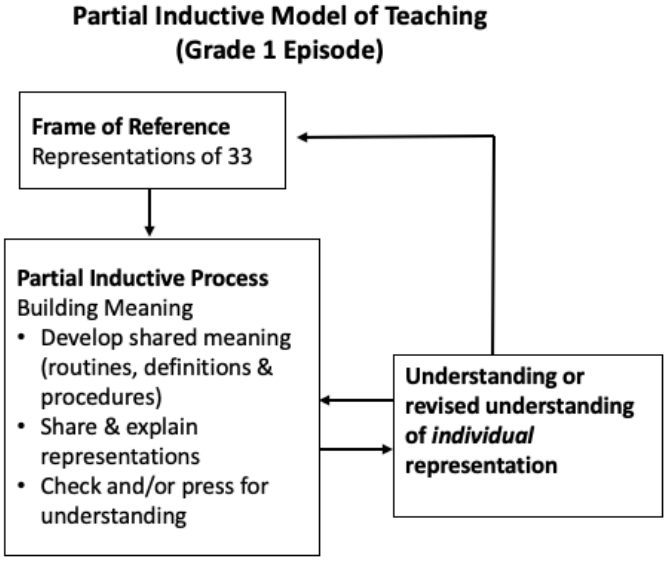


Models of teaching built from multilevel analysis of discourse can demonstrate pathways used and also suggest *potential* pathways for promoting discourse on a continuum of univocal to dialogic. For this study, an inductive model of teaching (Truxaw & DeFranco, 2008) was compared with models developed from analysis of math lessons in linguistically diverse classrooms. Thus far, fully inductive models of teaching have not been uncovered within the observed mathematics lessons. However, as noted in results of RQ1, there were instances of discourse that shifted somewhat toward dialogic and could be used to develop partial inductive models of teaching. For instance, a partial inductive model of teaching was developed from the episode related to the representations for the "number of days in school" from the first-grade classroom previously highlighted.

The partial inductive model of teaching (see Figure 9) developed from the first-grade classroom shows a *frame of reference* (representations of 33); a *partial inductive process* that includes developing shared meaning, sharing and explaining representations, and checking and/or pressing for understanding; and potential for *understanding or revised understanding of individual representations*. This model has some inductive tendencies (with potential for dialogic discourse) in that it includes opportunities to explain and press for understanding; however, it is only partially inductive or dialogic because it focuses predominantly on individual representations rather than moving toward more general mathematical ideas.

Figure 9

Partial Inductive Model Developed from Grade 1 Lesson



Compare this partial inductive model with the inductive model (refer back to Figure 3). Each has a frame of reference and some opportunity to explain and revise thinking. The inductive model, however, has a greater emphasis on revisiting the big mathematical ideas to develop connections and meaning *beyond individual problems*. The partial inductive model includes potential for cycles, but they do not build on each other meaningfully. For example, as students shared representations of the number 33, the individual representations were explained and the teacher pressed for understanding through use of generative assessment, such as "¿Por qué no es igual?" (Why isn't it equal?). The exchanges included dialogue with some shifts toward dialogic; however, rather than building on the larger conceptual ideas, the teacher moved on to different representations. In an inductive model the big ideas are revisited so that the students have greater potential to make connections and move beyond the individual representations or problems toward more general mathematical ideas. The partial inductive model demonstrates dialogic *potential*, but that potential is not fully realized.

Summary of RQ2. The analysis demonstrates that models of teaching can be developed through multilevel analysis of discourse moves, exchanges, sequences, and episodes within linguistically diverse mathematics classes. Thus far, the analysis has not revealed examples of fully inductive models of teaching within the focus classrooms; however, partial inductive models demonstrating potential for dialogic discourse have been developed.

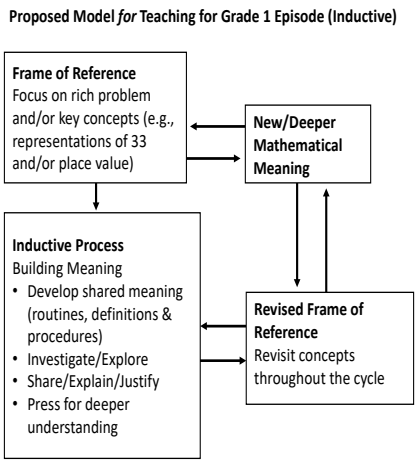
Moving toward an inductive model and dialogic discourse. Although an inductive model of teaching has not yet been uncovered within the observed math lessons, the partial inductive model provides insights into how teachers in linguistically diverse classrooms might press more toward inductive processes and dialogic discourse. Figure 10 shows a potential revision to the partial inductive model of teaching from the first-grade math lesson. This is a proposed model *for* teaching rather than a model of what occurred. This adjusted model suggests the importance of beginning with a more powerful frame of reference (e.g., with connections to mathematical concepts) that is revisited and revised dialogically as part of the teaching–learning process. The point is not that this revised model is the only way to support dialogic discourse. Rather, this example of a revised model suggests that it may be useful for teachers to consider inductive cycles of learning, along with verbal moves (e.g., generative assessment/feedback, exploratory talk, and accountable talk) and supporting moves (e.g., think time, use of L1, visuals, and code-switching). Considering not only moves and strategies but also cycles of learning may more powerfully support dialogic discourse in linguistically diverse mathematics classrooms.

Conclusions and Implications

Sociocultural theory suggests the need to recognize and support not only current competencies of students but also their potential competencies. For ELs, this means that, along with focus on vocabulary and procedures, they need opportunities to engage in rich mathematical discourse where they "are learning to describe patterns, make generalizations, and use representations to support their claims" (Moschovich, 2013, p. 46). This study identifies verbal moves associated with such discourse and suggests promising practices for supporting ELs. Specific verbal moves (e.g., exploratory talk, accountable talk, and generative assessment) accompanied by other supporting moves (e.g., think time, visuals, use of L1, and code-switching) may shift discourse toward dialogic and reinforce rich mathematical practices in linguistically diverse mathematics classrooms.

Furthermore, this study reveals that although attention to verbal and supporting moves is valuable, these are not sufficient for facilitating dialogic discourse in linguistically diverse classrooms. Attending to discursive *models of teaching* may provide a next step for productively challenging and supporting ELs. For example, a partial inductive model of teaching demonstrates some shifts toward dialogic discourse within a given episode but also reveals limitations in terms of broader mathematical meaning making. An inductive model provides more opportunities for empowering students to develop their own meaning about mathematics. An inductive model begins with a rich problem or task that provides a frame of reference for the mathematical ideas, then uses verbal and supporting moves to facilitate *discursive*

Figure 10
Proposed Model for Teaching



cycles that allow students to build understanding, investigate, share, explain, justify, revisit, revise, and develop meaning about mathematical ideas. Although inductive models of teaching have not been observed in mathematics lessons documented in this research study, the existence of the partial inductive model suggests that it is possible for teachers to push toward dialogic discourse in linguistically diverse classrooms. This seems to be a worthwhile goal.

The goals of this study are aligned with work suggesting pedagogical shifts in bilingual education that recognize *language in action*. "If teachers first rethink language and then think beyond language, they can begin to develop the kinds of robust understandings of language and bilingualism that will better position them to construct rich and equitable learning spaces" (Palmer & Martínez, 2013, p. 289). For mathematics educators who work in linguistically diverse classrooms, attention to discourse and associated models of teaching may position them to empower their ELs toward meaningful mathematical discourse, practices, and learning.

Although the small-scale nature of this study limits its generalizability, it demonstrates discourse analysis techniques and models that can be used to more deeply understand and enhance discourse in linguistically diverse mathematics classroom. Thus the research provides potential to empower students in linguistically diverse elementary mathematics classrooms.

Note

[1] All names are pseudonyms.

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